Abstract

In the next few decades we may develop AI that can automate ~all cognitive tasks and dramatically accelerate economic growth. By contrast, today the capabilities and impact of AI are much more limited. Once we have AI that could readily automate 20% of cognitive tasks (weighted by 2020 economic value), how much longer until it can automate 100%? This is what I refer to as the question of AI takeoff speeds; this report develops a compute-centric framework for answering it. First, estimate how much more “effective compute” – a measure that combines compute with the quality of AI training algorithms – is needed to train AI that could readily perform 100% of tasks compared to AI that could just perform 20% of tasks; my best-guess is 4 orders of magnitude more (i.e. 10,000X as much). Then, using a computational semi-endogenous growth model, simulate how long it will take for the effective compute used in the largest training run to increase by this amount: the model’s median prediction is 3 years. The simulation models the effect of both rising human investments and increasing AI automation on AI R&D progress. It predicts that the transition from full automation of AI R&D to superintelligence will happen in 1 - 12 months.

How to read this report

Read the short summary. Then play around with the Full Takeoff Model here.

Then, if you have a background in growth economics, or are particularly mathsy, read this concise mathematical description of the Full Takeoff Model.

Then read the long summary. At the end, I list some particular sections of the full report that I think would be most useful to read next.

In general, I do not recommend reading the full report top to bottom. Instead I’d treat its sections as providing longer discussions of the values of important modelling assumptions and parameter values.

Or, instead of any of the above, you could read this high level informal discussion of the main qualitative arguments and evidence contained in the report. This is less technical but touches upon the important points. I’d recommend this first for most readers.
0. Short summary (~5 pages)

In the next few decades we may develop AGI that could readily\(^1\) perform ~all cognitive tasks as well as a human professional. If we can run enough AGIs, they could ~fully automate cognitive labour and dramatically transform the world.

By contrast, today the capabilities and impact of AI are much more limited. How sudden might this transformation be? More precisely: once AI is capable enough, and we can run enough copies, that AI can readily automate 20% of cognitive tasks (weighted by 2020 economic value), how much longer until AI can readily automate 100%?\(^2\) This is what I refer to as the question of AI takeoff speeds.\(^3\)

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1 The phrase “readily” here indicates that i) it would be profitable for organisations to do the engineering and workflow adjustments necessary for AI to perform the task in practice, and ii) they could make these adjustments within 1 year if they made this one of their priorities.

2 Milestones of the form “AI could readily automate \(x\)% of tasks” require both that AI is capable enough to perform the tasks and that we can run enough copies for AI to replace every human doing those tasks. By contrast, the definition of AGI as “AI that could readily perform 100% of tasks” only requires that we could run one copy.

3 By contrast, in Superintelligence takeoff speed is defined as the time from human-level AI to superintelligence. I discuss this, but it is not my focus because I think the bigger crux in takeoff speeds relates to the time from sub-human but hugely impactful AI to human-level AI (more).
This report develops a framework to estimate takeoff speeds, extending the biological anchors framework (Bio Anchors) for predicting when we’ll develop AGI (AI that could readily perform ~100% of cognitive tasks). It also has implications for AGI timelines.

I use a compute-centric framework in which:

- Some amount of compute would have been sufficient to train AGI using ideas and algorithms from 2020. The exact amount is highly uncertain and very large compared to today’s biggest training runs.
- Software progress, by which I mean improvements in ideas and algorithms for training AI systems, merely decreases the compute required to train AGI. Or, equivalently, software progress increases the amount of “effective compute” we have to train AGI.
  - Effective compute = software * physical compute.
  - So one FLOP training AIs with 2025 software may be worth ten FLOP with 2020 software.
  - All forms of algorithmic progress are represented as increasing the quantity of effective compute per unit of physical compute.

In this framework, AGI is developed by improving and scaling up approaches within the current ML paradigm, not by discovering new algorithmic paradigms.

Within this compute-centric framework, I first estimate the 'capabilities distance' we need to traverse during takeoff. Second, I calculate the 'speed' at which we will acquire those capabilities by simulating a macroeconomic model of software R&D, hardware R&D, and increasing spending on AI training runs. Then takeoff time \(\sim \frac{\text{distance}}{\text{average speed}}\).

More precisely, by 'distance' I mean: How much more effective compute do you need to train AI that can readily perform ~all cognitive tasks (AGI) compared to weaker AI that can only readily perform 20% of these tasks (weighted by 2020 economic value)? I call this the effective FLOP gap you need to cross during AI takeoff. Its size is very uncertain, but we can make guesses weakly informed by evidence from ML and biology. My best guess is that the effective FLOP gap is \(~4\) orders of magnitude (OOMs): we’ll need \(10^4\) times as much effective compute to train AGI as to train AI that can readily perform 20% of cognitive tasks. But anything from \(~1\) OOM to \(~8\) OOMs seems plausible.

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4 The explicit forecasting target of Bio Anchors is “transformative AI”, but the framework can be used to forecast AGI.
5 More precisely, if experts in 2020 were given ~2-5 years to experiment with the new quantity of compute and design a training run, they could train AGI (more).
6 This is a slight oversimplification. Takeoff time as defined here is time from AI that could readily perform 20% of tasks to AI that could perform 100%. Whereas the metric I ultimately report is AI could readily automate 20% to 100%. The latter accounts for whether we have enough compute to run enough copies to automate those tasks in practice, given the size of the human population. In practice though, this mostly makes very little difference to the results (<1 year) because runtime compute is rarely a bottleneck.
7 Why 20%? The choice of startpoint is fairly arbitrary. 5% would be too low, as it could be driven by a few one-off wins for AI like “automating driving”; 50% would be too high as by then AI may already be having ~transformative impact.
By our ‘speed’ crossing the effective FLOP gap I mean: how quickly will we increase the effective compute used in the largest training run?

We can cross the gap by increasing three quantities:
1. The quality of AI software, i.e. algorithms for training AI. If the level of software doubles, we get twice as much effective compute for each FLOP.
2. The quality of AI hardware, measured as FLOP/$. Improved hardware allows us to buy more FLOP with a fixed budget.
3. $ spend on FLOP in the largest training runs.

These three quantities multiply together to give the effective compute in the largest training run:

\[ \text{Effective compute in the largest training run} = \text{software} \times \text{FLOP/$} \times \text{$ on FLOP}. \]

This implies we can calculate our speed crossing the effective FLOP gap as:

\[ g(\text{effective compute in the largest training run}) = g(\text{software}) + g(\text{FLOP/$}) + g(\text{$ on FLOP}). \]

First estimate the size of the effective FLOP gap; then calculate how quickly we’ll cross it by simulating the trajectories of {$ on FLOP in the largest training run}, FLOP/$ and software. As we cross the effective FLOP gap, AI automates more tasks and so AI R&D progress accelerates.

I use a computational model, the Full Takeoff Model, to calculate the evolution of software, FLOP/$ and {$ on FLOP in the largest training run}. The Full Takeoff Model is designed to capture the most important effects from:

I. **Rising human investments** in software R&D, hardware R&D and AI training runs.
   - I use semi-endogenous growth models to predict how R&D spending will translate into software and hardware progress.

II. **Increasing AI automation** of software R&D, hardware R&D, and the broader economy.
   - I use CES task-based models to predict how AI automation affects software R&D progress, hardware R&D progress, and GDP.

[Link to diagram.]
○ The model implies that software, hardware and GDP grow increasingly quickly as we cross the effective FLOP gap and AI automates more tasks.
○ I assume it is somewhat easier for AI to automate cognitive tasks in {software and hardware R&D} than in the broader economy. This makes takeoff faster.

The Full Takeoff Model makes assumptions about the compute needed to train AGI using 2020 algorithms, the size of the effective FLOP gap, the pace at which human investments rise, the diminishing returns to hardware and software R&D, bottlenecks from tasks that AI cannot perform, and more. It calculates trajectories for software, hardware, $ on training, effective compute in the largest training run, and GDP.

9 Link to chart.
Simulation of Full Takeoff Model with my best-guess values for each parameter.
In the playground you can enter your preferred parameter values and study the results.
($ on FLOP in largest training run $ on FLOP globally * fraction of global FLOP on training)

Monte Carlo analysis

We perform a Monte Carlo analysis to get a distribution over takeoff speed given our uncertainty about these parameters:

<table>
<thead>
<tr>
<th>MC results</th>
<th>Takeoff speed[^10]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Years from “AI could readily automate 20% of cognitive tasks” to “AI could readily automate 100% of cognitive tasks”.[^11]</td>
</tr>
<tr>
<td>Percentile</td>
<td>Tasks in the general economy.</td>
</tr>
<tr>
<td>1%</td>
<td>0.3</td>
</tr>
<tr>
<td>10%</td>
<td>0.8</td>
</tr>
<tr>
<td>20%</td>
<td>1.2</td>
</tr>
</tbody>
</table>

[^10]: Conditional on 100% automation before 2100.
[^11]: Reminder: milestones of the form “AI could readily automate x% of tasks” require both that AI is capable enough to perform the tasks and that we can run enough copies for AI to replace every human doing those tasks. By contrast, the definition of AGI as “AI that could readily perform 100% of tasks” only requires that we could run one copy.
[^12]: Why is takeoff slower for AI R&D tasks? The Monte Carlo puts weight on AI R&D being significantly easier to automate than the general economy. If so then, by the time AI can readily automate 20% of tasks in the general economy, it will have already automated most tasks in AI R&D and significantly accelerated AI progress. This factor speeds up takeoff for tasks in the general economy but not for tasks in R&D.
There is a strong relationship between the difficulty of developing AGI and takeoff speed. If AGI is easier to develop then (in expectation):

1. The effective FLOP gap is smaller, because it is bounded from above by AGI training requirements.\(^1\)

2. Our average speed crossing it is higher:
   a. One way to quickly cross the effective FLOP gap is to quickly increase the fraction of the world’s computer chips used on the largest training run.
   b. If AI is easy to develop, this fraction will still be small when we start crossing the effective FLOP gap. So there’s more room to grow the fraction as we cross the gap.
   c. If in addition the effective FLOP gap is small (point 1), we could cross most of the gap merely by increasing the fraction. I.e. we could cross the gap very quickly.

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\(^1\) If AGI could be trained with X OOMs more effective compute than today’s largest training run, the effective FLOP gap must be <X OOMs.
To get to my personal all-things-considered bottom line, I eye-ball adjusted the Monte Carlo results for incorporate limitations of the model, especially i) ignoring types of discontinous progress around AGI that can’t be represented by a narrow effective FLOP gap and ii) not modelling various lags to training and deploying advanced AI.

My personal probabilities are still massively in flux, but my current best guesses are:

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Takeoff speed</th>
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</thead>
<tbody>
<tr>
<td>3%</td>
<td>0.1</td>
</tr>
<tr>
<td>10%</td>
<td>0.3</td>
</tr>
<tr>
<td>20%</td>
<td>0.8</td>
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<tr>
<td>50%</td>
<td>3</td>
</tr>
<tr>
<td>80%</td>
<td>10</td>
</tr>
<tr>
<td>90%</td>
<td>20</td>
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</tbody>
</table>

### How much time from AGI to superintelligence?

This has not been my main focus, but the framework has implications for this question. My best guess is that we go from AGI (AI that can perform ~100% of cognitive tasks as well as a human professional) to superintelligence (AI that very significantly surpasses humans at ~100% of cognitive tasks) in 1 - 12 months. The main reason is that AGI will allow us to >10X our software R&D efforts, and software (in the “algorithmic efficiency” sense defined above: effective FLOP per actual FLOP) is already doubling roughly once per year.

### Implications for AI timelines

Compared to Bio Anchors, this framework predicts larger maximum $ spend on the largest training for AGI, includes additional speed-ups from AI automation, and models the possibility we could leverage enormous amounts of runtime compute to get full automation sooner. As a result, its median predicted AGI year is 10 years earlier than Bio Anchors (2043 vs 2053), despite using the same distribution over AGI training requirements.

14 Reminder: milestones of the form “AI could readily automate x% of tasks” require both that AI is capable enough to perform the tasks and that we can run enough copies for AI to replace every human doing those tasks. By contrast, the definition of AGI as “AI that could readily perform 100% of tasks” only requires that we could run one copy.
Notable assumptions and limitations of the framework

- Compute-centric framework that assumes that some amount of compute would be sufficient to train AGI in 2020, and that algorithmic progress merely reduces that amount.
- Models software and compute as inputs to AI development, but not data/environments.
- Assumes that AI capabilities improve continuously with software research effort (research into AI algorithms). More.
- Ignores lags between developing and deploying AIs, and ignores other delays. More.
- I mostly focus on the transition to AGI rather than the aftermath of AGI, as this is where I think the origin of fast takeoff is most likely to be. More.
- Cannot directly make predictions about many strategically-important AI milestones.
  - It can predict “AI can readily add $10tr/year to GDP”, “AI can readily automate 30% of R&D”, “GDP is growing at 30%/year”, or “AI cognitive output exceeds that of 10 billion humans”.
  - It cannot predict “AI has situational awareness”, “AI is super-human at persuasion/deception”, or “AI kills us all if it’s not aligned”. Though these are plausibly correlated with things the framework can predict.

I recommend playing around with the FTM before reading the summary.

I recommend that readers familiar with economic growth theory now read a concise mathematical description of the full economic model.
Acknowledgements

- To Carl Shulman especially for introducing many of the core ideas.
- To Jaime Sevilla, Eduardo Infante Roldán, and others at Epoch for coding up the Full Takeoff Model, creating the playground and many helpful suggestions.
1. Long summary (~25 pages)

What is takeoff speed?

Roughly speaking, I focus on the amount of calendar time between “AI is capable enough to have massive economic impact” and “AI is that is completely transformative”.

I’m not aware of a very principled way to specify the startpoint and endpoint here.

One option is economic growth rates, e.g. years from GDP growing at 5%/year to 30%/year. This is easily measurable, but has the downside that AI capabilities might grow explosively but have little, or very delayed, impact on GDP due to various bottlenecks.

AGI can be defined as AI that can perform ~100% of cognitive tasks as well as a human professional, and a second option to extend that definition to less capable AI. In particular, quantify AI capabilities via the % of cognitive tasks AI can readily perform, where each task is weighted by its economic value in 2020. (The phrase “readily” here indicates that i) it would be profitable for organisations to do the engineering and workflow adjustments necessary for AI to perform the task in practice and ii) they could make these adjustments within 1 year if they made it one of their priorities.) This second option also dovetails nicely with economic models of automation, but it has a few weaknesses.

A third option is the % of cognitive tasks AI can readily automate. This is like the second option but comes with the additional requirement that we can run enough copies of the AI(s) to actually replace the humans currently performing the tasks. This is more relevant to the collective cognitive capacity of AI systems. Today, valuations of AI market size are $10-100b, suggesting that AI can readily automate <1% of cognitive tasks.

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15 This could be one generally capable AI, or many narrow AIs working together. I think the takeoff model I ultimately use sits better with the latter interpretation.

16 By “cognitive task” I mean “any part of the workflow that could in principle be done remotely or is done by the human brain”. So it includes ~all knowledge work but also many parts of jobs where you have to be physically present. E.g. for a plumber it would include “processing the visual and audio inputs relating to the problem, choosing a plan to solve it, and deciding what specific actions to take second by second”. But it doesn’t include the “pure physical labour” parts of plumbing that require a physical body.

17 First, it will be hard to measure in practice what % of cognitive tasks AI can perform (more). Second, there are not literally a fixed set of tasks in the economy; new tasks are introduced over time and AI will contribute to this (more, more). Thirdly, at what cost can AI perform these tasks? The model implies it will be able to perform them very cheaply, more cheaply than humans (more). Fourth, whether AI can perform a task may depend on whether ‘nearby’ tasks have already been automated, but I don’t model this (more).

18 E.g. here, here, here, here. I don’t know how reliable these estimates are, or even their methodologies.

19 AI’s impact on GDP may be greater than its market size, but it would have to have to be adding ~$0.5tr/year to GDP to have automated 1% of cognitive tasks.
The full sensitivity analysis includes a variety of ways of quantifying takeoff speed, but I currently focus on years from “AI could readily automate 20% of cognitive tasks” to “AI that could readily automate 100% of cognitive tasks”.

- The startpoint corresponds to AI that could readily increase world GDP by ~$10tr/year, assuming AI was deployed wherever it was profitable.
  - Standard Baumol effects will diminish the fraction of GDP paid to the automated tasks after they are automated, but this is still the effect on total GDP.
- The endpoint corresponds to AI that could collectively replace all human cognitive output. I believe that by this point we very likely have AI that could permanently disempower all of humanity if it wanted to.

Calculating takeoff speed

What’s driving the results on a high level?

My process for estimating takeoff speed is as follows:

- First estimate the **effective FLOP gap**.
  - Roughly speaking, this is the “difficulty gap” from AI that can readily perform 20% of cognitive tasks to AGI. How much harder is the latter to develop than the former?
  - Within our compute-centric framework, we translate “How much harder?” to “How much more effective compute is required during training?”.
  - More precisely, the effective FLOP gap means: How much more effective compute do you need to train AGI compared to AI that can only readily perform ~20% of cognitive tasks (weighted by 2020 economic value)?
  - Result: the effective FLOP gap is ~1 - 8 OOMs, best guess ~4 OOMs.
- Use a **toy model** to estimate how quickly we’d cross the effective FLOP gap from human investments alone, ignoring the effects of AI automation.
  - This can be roughly interpreted as “How fast would takeoff be if no one ever actually deployed the AIs in the real world, and just used them for demos?”
  - Result: we initially increase the effective compute in the largest training run by ~0.7 OOMs/year, then this falls to ~0.4 OOMs/year as we can no longer increase the fraction of the world’s AI chips on the largest training run. In my best-guess

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20 Why 20%? The choice of startpoint is fairly arbitrary. 5% would be too low, as it could be driven by a few one-off wins for AI like “automating driving”; 50% would be too high as by then AI is already having ~transformative impacts.

21 World GDP is ~$100tr, about half of which is paid to human labour. If AI automates 20% of that work, that’s worth ~$10tr/year.

[This is a bit high, as many tasks have a component of physical labour (though all have some cognitive component). On the other hand, AI will probably produce more output at those tasks than the humans they replace (as they’re cheaper to run), increasing their value-add; and AI will additionally increase the rates of capital accumulation and of tech progress.]

22 More precisely, deployed anywhere where i) it would be profitable for organisations to do the engineering and workflow adjustments necessary for AI to perform the task in practice and ii) they could make these adjustments within 1 year if they made this one of their priorities.
scenario, with a gap of 4 OOMs, I get an average speed of \(~0.5 \text{ OOMs/year}\) and so takeoff takes \(~8\text{ years}\).

- At this point we’re assuming that soon after we get \textit{AI that can readily perform x\% of tasks}, we can run enough copies that \textit{AI could readily automate x\% of tasks}. This turns out to be an OK simplification.

- Simulate the Full Takeoff Model (FTM) to calculate how quickly we will cross the effective FLOP gap in the best-guess scenario. The FTM:
  - Models the effects of human investment more carefully, including some additional bottlenecks and complications that the toy model ignored.
    - \textbf{Result:} this slows takeoff in the best-guess scenario to \(~12\text{ years}\).
  - Uses a \textit{standard model of automation} from growth economics to predict the effect of partial AI automation on software progress, hardware progress, and GDP (and thus $ spend on training runs).
    - As the \textit{effective compute used in the largest training run} increases, AI automates more and more tasks.
    - \textbf{Result:} this speeds up takeoff in the best guess scenario to \(~5\text{ years}\).
  - The FTM keeps track of how many compute we have for running AIs, so it can calculate the desired “AI could readily automate x\% of tasks” metric.

- Run a Monte Carlo simulation to get a probability distribution over takeoff speed.
  - The incompatibility between low AGI training requirements and large FLOP gaps lowers the median sampled effective FLOP gap to \(~3.3\text{ OOMs}\). This speeds up median takeoff to \(~4\text{ years}\).
  - The possibility that it’s easier for AI to automate \{hardware and software R&D\} than the general economy reduces the median takeoff to \(~3\text{ years}\).
  - Takeoff speed in the \[10th, 50th, 90th\] percentiles of the Monte Carlo are \[~[0.8, 3, 11]\text{ years}\].
    - The <1 year takeoff is mostly driven by:
      - Small effective FLOP gap + large scope for increasing the fraction of chips used for the largest training run \(\rightarrow\) you can quickly cross the gap by increasing spending on the largest training run.
      - The possibility of significant AI R&D automation by the time we start crossing the gap.

To summarize:

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Takeoff speed</th>
</tr>
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<tbody>
<tr>
<td>Toy model without AI automation</td>
<td>8 years = 4 OOMs / 0.5 OOMs per year</td>
</tr>
<tr>
<td>Full Takeoff Model with best-guess inputs</td>
<td>5 years</td>
</tr>
<tr>
<td>Median result from the Monte Carlo</td>
<td>3 years</td>
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</table>

The next three sections discuss the size of the effective FLOP gap, the toy model that excludes AI automation, and the Full Takeoff Model’s treatment of AI automation.
The effective FLOP gap is ~1 - 8 OOMs, best guess ~4 OOMs

How much more effective compute do you need to train AGI than to train AI that can readily perform ~20% of cognitive tasks (weighted by 2020 economic value)?

Hans Moravec’s “rising tide of AI capacity” can illuminate the meaning of the effective FLOP gap. Currently AI can only readily perform a small fraction of cognitive tasks – the areas of the map that are underwater. Over time the AI capabilities improve and the tide rises. Eventually, AI can readily perform all cognitive tasks: we’ve crossed the effective FLOP gap and everything is under water.

The size of the effective FLOP gap is very uncertain, but evidence from biology and ML can weakly inform the choice. Here’s a brief summary of some key pieces of evidence.

- **AGI training requirements** place an upper bound on the effective FLOP gap.
  - E.g. if AGI can be trained with 1e36 FLOP with 2020 algorithms (my median) and we’ve already done a 3e24 FLOP training run, the gap must be <7.5 OOMs.
- **Limitations of SOTA AI** tighten this bound.
  - I think a 300X scale-up of today’s SOTA AI (1e27 effective FLOP) wouldn’t be sufficient to readily perform 20% of cognitive tasks, in which case the effective FLOP gap must be <9 OOMs.
- **How AI capabilities vary with training FLOP: suggests ~5 OOMs**
  - GPT-3 had a ~300X bigger training run than GPT-2; you can play around with both to get a feel for their capabilities. I guess you might need two equally big improvements to cross the effective FLOP gap, which implies it’s ~5 OOMs.
  - GPT-N looks like it will solve some LM benchmarks ~4 OOMs earlier than others (see diagrams in link); I’d guess the effective FLOP gap will be bigger than this as economic tasks are much more varied along many dimensions.
- **How human + animal capabilities vary with brain size: suggests ~2 OOMs**
  - Data suggest a 10% bigger brain (in terms of FLOP/s) grants ~5 extra IQ points. Extrapolating heroically, a 10X bigger brain grants ~120 IQ points, which seems sufficient to cross the effective FLOP gap.
    - Anchoring to model size, a 10X bigger model might require 100X more training FLOP, suggesting a gap of ~2 OOM.
    - Anchoring to lifetime learning compute, a 10X bigger brain requires 10X more lifetime learning FLOP. So you might see the same improvement in ML by increasing training FLOP by 10X → gap of ~1 OOM.
    - You could get smaller gaps by using smaller IQ point gaps.
  - You can make similar arguments via chimp/human comparisons.
  - Importantly, this approach ignores the fact AI may have strong comparative advantages over humans on some tasks but not others, allowing very “limited” AIs to automate many tasks.

- **Practical difficulties with partially automating jobs: suggests a smaller effective FLOP gap.**
  - For AI to be able to readily perform 20% of cognitive tasks, it must be able to do so without too much additional engineering work or rearranging of workflows.\(^\text{23}\)
  - If there are significant practical difficulties in partially automating jobs,\(^\text{24}\) then the capabilities for non-trivial partial automation may only be a little lower than those for full automation.
  - We can incorporate this by using a smaller effective FLOP gap than we otherwise would have.

- **Horizon length.**
  - If “short horizon”\(^\text{25}\) training can perform 20% of cognitive tasks but performing all cognitive tasks requires “long horizon” training, the effective FLOP gap will be >5 OOMs.\(^\text{26}\)

<table>
<thead>
<tr>
<th>Evidence</th>
<th>Effective FLOP gap estimate</th>
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<tbody>
<tr>
<td>AGI training requirements + Limitations on SOTA AI</td>
<td>~&lt;9 OOMs</td>
</tr>
<tr>
<td>How AI capabilities vary with training FLOP</td>
<td>~5 OOMs</td>
</tr>
<tr>
<td>How human + animal capabilities vary with brain size</td>
<td>~2 OOMs</td>
</tr>
</tbody>
</table>

\(^{23}\) Reminder: if AI can “readily” perform a task then i) it would be profitable for organisations to do the engineering and workflow adjustments necessary for AI to perform the task in practice, and ii) they could make these adjustments within 1 year if they made this one of their priorities.

\(^{24}\) E.g. Brynjolfsson (2018) finds that “most occupations include at least some [automatable by machine learning] tasks; (iii) few occupations are fully automatable using ML; and (iv) realizing the potential of ML usually requires redesign of job task content.

\(^{25}\) This concept is from Bio Anchors. Short horizons means that the model only needs to “think” for a few seconds for each data point; long horizons means the model needs to “think” for months for each data point and so training requires much more compute.

\(^{26}\) There are ~5 OOMs between a “10 second” horizon length and a “1 month” horizon length.
Practical difficulties with partially automating jobs | Shorter gap
---|---
Horizon length | >5 OOMs
Overall | ~4 OOMs (~1 to 8 OOMs)

All in all, my best-guess is that the effective FLOP gap is ~4 OOMs, with values from ~1 to ~8 OOMs possible. Lower than ~2 OOMs feels out-of-whack with how SOTA AI abilities scale with training FLOP and with the plausibility of AI having strong comparative advantages on certain tasks; much higher than ~5 OOMs feels in tension with how IQ scales with brain size in humans and chimps.

More on evidence about the size of the FLOP gap.

Speed crossing the effective FLOP gap from human investment, ignoring AI automation

Recall, we cross the effective FLOP gap by increasing the effective compute used in the largest training run, which can be calculated as

\[ \text{Effective compute in the largest training run} = \text{software} \times \text{FLOP/$} \times \text{$ on FLOP}. \]

This implies we can calculate our speed crossing the effective FLOP gap as:

\[ g(\text{effective compute in the largest training run}) = g(\text{software}) + g(\text{FLOP/$}) + g(\text{$ on FLOP}). \]

This section estimates how quickly each of these three components will grow due to rising human investments as we approach AGI.

Background - “waking up” to advanced AI’s economic potential

I believe pre-AGI systems have the potential to increase world GDP by $10s of trillions per year. By contrast, AI software and hardware investments are currently in the $10s billions. So I expect
investments in AI to grow more rapidly once relevant actors “wake up” to the economic and strategic potential of AI.\(^{27}\)

Let’s look at the effect of human investment on each component in turn.

**More $ spent on FLOP for the largest training run**

I split this into two sub-components:

\[
\text{\$ on FLOP for the largest training run} = \text{\$ on FLOP globally} \times \text{fraction of global FLOP on the largest training run}
\]

I guess that \textit{$ on FLOP globally$} will double every \(~3\) years, based on evidence about recent semiconductor revenue growth, the time to build a fab, and the expansion of munitions production at wartime.\(^{28}\) More.

I estimate that there’s currently room to increase the \textit{fraction of global FLOP on the largest training run} by \(~3\) - \(~4\) OOMs, but that this will decrease to \(~1\) - \(~2\) OOMs by 2030. After “wake up”, I guess the fraction will increase by \(~3\)X per year (growth rate of \(72\%\)) until it hits a cap. This quickly moves us through part of the effective FLOP gap, and then stops having an effect. More.

I forecast faster growth here than Bio Anchors, shortening timelines. Bio Anchors forecasts \textit{$ on FLOP for the largest training run$} to only grow at \(~3\)% after reaching \(~$200b\), whereas I expect it to continue to double every \(~3\) years (growth rate of \(~22\%)\) after “wake up” until it caps out at \(~1\)% of world GDP (though in simulation, we typically get AGI before training runs are this big).

**More FLOP/$ from better quality hardware**

Bio Anchors directly extrapolates past trends in FLOP/$, predicting a 2.5 year doubling time. By contrast, I fit a semi endogenous growth model to historical data about how hardware R&D spending translates into more FLOP/$, predict future R&D spending, and then calculate future FLOP/$.

The fitted model suggests that \textit{each doubling of cumulative hardware R&D spending drives \(~5\) doublings of FLOP/$}. So if cumulative spending grows at \(x\)%, FLOP/$ is predicted to grow at \(5x\)%\(^{29}\).

---

\(^{27}\) This could be either AI organisations reinvesting revenues from AI products, or impressive demos attracting external investment. I don’t model either in detail and instead just try to ballpark the overall rate of investment growth.

\(^{28}\) This, like FLOP production as we approach AGI, is an example of “growth of a specific industry’s output when there is suddenly very large demand”.

\(^{29}\) This assumes the growth rates are \textit{instantaneous} growth rates, defined as \(e^{gt}\). All the growth rates I report are instantaneous.
Recently the growth rate of cumulative spending has only been ~5%, and annual hardware R&D spending is <$100b. This suggests there’s plenty of room for spending to grow after “wake up”.

I guess that annual R&D spending will grow at ~17% (~4 year doubling) after “wake up”. This is based on eye-balling historical growth of hardware R&D, growth of R&D in other areas, and growth in US defense and space R&D after WW2.\(^{30}\)

If cumulative R&D spending were growing at 17%, I’d predict that FLOP/$ would grow 17*5 = 85% (~0.8 year doubling). But it turns out that if annual spending suddenly switches from 5% to 17% growth, then growth of cumulative spending rises gradually from 5% to 17%, only exceeding 10% after ~8 years.\(^{31}\) So my current forecast is that, after “wake up”, FLOP/$ initially doubles every ~3 years but grows increasingly quickly over time.\(^{32}\)

This forecast feels a little slow. It’s plausible that annual R&D spending has a quick one-off increase of >2X before growing more slowly, and this would imply faster growth of FLOP/$ with less delay. On the other hand, I’m not sure the R&D sector could productively absorb that much money, and I’m forecasting the growth of quality adjusted R&D inputs.

More on the effects of human investment on FLOP/$.

Better software

My process for software is the same as for hardware: I fit a semi endogenous growth model to historical data about how software R&D spending translates into better software, predict future software R&D spending, and then calculate future software.

There’s massive uncertainty in the historical data about the rate of software improvement. Most measurements of software progress in specific domains suggest the quality of algorithms doubles every ~1-2 years; I follow Bio Anchors’ in making the conservative assumption that progress for AGI training algorithms is slower, with a doubling time of only 2.5 years. A more aggressive assumption here would reduce AGI timelines by 2-5 years and speed up takeoff.

Combined with a shaky estimate of the growth of software R&D spending, the fitted model implies that each doubling of cumulative software R&D spending drives ~1.25 doublings of software.

I forecast that cumulative spending will grow at ~25% after “wake up”, implying that software will grow at 25*1.25 = 31%, a ~2.2 year software doubling time.

\(^{30}\) This, like hardware R&D as we approach AGI, is an example of “growth of a specific industry’s R&D when there is suddenly very large demand”.

\(^{31}\) This sheet illustrates the dynamic.

\(^{32}\) The Full Takeoff Model caps total hardware R&D spending at 1% of global GDP.

\(^{33}\) The report operationalises “a doubling of software” to mean: your algorithms now use physical FLOP twice as efficiently. So doubling software has exactly the same effect as doubling your quantity of physical FLOP.
More on the effects of human investment on software.

Total time to cross the effective FLOP gap from human investment

So, combining the above, I calculate the growth of the the largest training run after “wake up” as follows:

$$g(\text{effective compute in largest training run}) = g(\text{$ on FLOP}) + g(\text{FLOP/\$}) + g(\text{software})$$

$$= g(\text{fraction FLOP on training run}) + g(\text{$ on FLOP globally}) + g(\text{FLOP/\$}) + g(\text{software})$$

$$= \text{Initially } \sim 75\%, \text{ then } \sim 0\% + \sim 22\% + \sim 25\% \text{ (increasing over time)} + \sim 31\%$$

$$= \text{Initially } \sim 153\%, \text{ then } \sim 78\% \text{ (increasing over time)}$$

$$= \text{Initially } \sim 0.7 \text{ OOMs/year}, \text{ then } \sim 0.3 \text{ OOMs/year (increasing over time)}$$

What might this imply about takeoff speed? If there’s a 4 OOM effective FLOP gap, and you can cross 1 OOM by increasing “the fraction” (of global FLOP used on the largest training run), then it will take you $\sim 8 \text{ years}$ to cross overall.\(^{35}\)

Note, if the effective FLOP gap was only 2 OOMs, you could cross it in just 2-3 years.\(^{36}\) So increasing “the fraction” allows you to cross short FLOP gaps especially quickly.

If you’re confused about how the different quantities discussed here combine together to give an estimate of takeoff speeds, I recommend looking at this toy model.

The parameters discussed in this section (the growth rates of human investment and the returns to hardware and software R&D) play important roles in the Full Takeoff Model.

\(^{34}\) Link to image.

\(^{35}\) Time = distance / speed = 3 OOMs to cross without the fraction / $[\sim 0.4 \text{ OOMs/year}] = \sim 8 \text{ years}$.

\(^{36}\) 1 OOM to cross with the fraction * 3 years per OOM = 3 years.
Speed crossing the effective FLOP gap, *including* effects from AI automation

The results of this section ultimately come from simulating the Full Takeoff Model, a economic growth model that combines the effect of human investment and AI automation to estimate how the effective FLOP on the largest training run changes over time.

The main thing the FTM adds to the analysis above is modelling AI automation of software R&D, hardware R&D, and GDP. So let's start by examining that part of the FTM.

AI automation increases our average speed crossing the effective FLOP gap by ~2.5X.

AI automation increases GDP and the amount of hardware and software R&D progress made each year. This causes the effective compute on the largest training run to grow increasingly quickly as we cross the effective FLOP gap.

To estimate the effect of continuously increasing AI automation I adapt a [task-based CES model](#) from the economic automation literature. Here’s a *very simplified toy version* of what my model says about how AI automation affects GDP. (Later I’ll introduce various complications.)

- At first, AIs perform <1% of economic tasks, reflecting current annual AI revenues being <1% of global GDP. The other tasks are performed by humans, so GDP is ~proportional to the number of humans. (There’s no capital in this toy example; it is in the FTM and I discuss it below.)

---

37. [Link to diagram.](#)
Over time, the effective compute in the largest training run increases. As a result, AIs automate an increasing fraction of economic tasks. This increases GDP. How much by?

- Let’s say AIs automate 50% of tasks. This boosts GDP in two ways.
  - Firstly, humans can focus on the remaining 50% of tasks, and AIs can match the per-task output of humans, increasing the output per task by 2X. This boosts GDP by 2X.  
  - Secondly, AIs soon have more output-per-task than humans, because there are many more AIs than humans. This increases GDP even further.
    - How much further depends on the extent to which GDP is bottlenecked by the unautomated human tasks.
    - I consider some weakly-relevant empirical evidence about the strength of these bottlenecks.
    - My best-guess for the bottleneck implies that GDP could rise a further 4X in this example, if you had unlimited AIs performing their 50% of tasks.
    - Exactly what the boost is depends on how many AIs you run to do the automated tasks. After “wake up”, I think actors will be willing to spend a lot of $ running AIs that can accelerate hardware and software progress, so I expect this number to be large.

So AI automating 50% of tasks increases GDP by ~2X - 8X.

- By analogous logic, automating 20% of tasks boosts GDP by ~1.2X - 2X; automating 75% of tasks boosts GDP by ~4X - 60X.

How do we decide how much automation has happened? The FTM has a mapping from {effective compute in the largest training run} to {% of cognitive tasks that AI can perform}.

In the best-guess scenario the mapping is such that:
- 1e36 effective FLOP → AI can perform 100% of cognitive tasks
- 1e32 effective FLOP → AI can perform 20% of cognitive tasks
- This matches the best-guess effective FLOP gap of 4 OOMs.

---

31 Automating a task requires both that AI can perform it and that there’s enough runtime compute to automate the task in practice (i.e. enough compute to cheaply replace the humans currently doing the task). It turns out that runtime compute is rarely a bottleneck to automation.  
32 Here I’m assuming that AI output-per-task at least keeps up with human output per task. I can assume this because it’s a condition for AI automating the tasks in the first place.  
33 If AI had automated x% of tasks, the boost here would equal 1/(1-x%).  
34 In the growth model, the strength of this bottleneck depends on the substitutability between different tasks: how much can more AI labour on task 1 make up for limited human labour on task 2?  
35 The Full Takeoff Model assumes that the percentage of global compute used to run AIs doing software rises quickly to a cap of ~20% after “wake up”.  
36 Here’s the formula. If AI automates x% of tasks, the minimum effect is 1/(1-x%); the maximum effect is 1/(1-x%)(1 + 1/-rho), where rho controls the strength of the bottleneck. My best guess is rho=0.5, so the maximum effect is 1/(1-x%)^3. 1/(1-0.2) = 1.25, 1.25^3=1.95; 1/(1-0.75) = 4, 4^3=64.  
37 I.e. an amount of effective compute equivalent to using 1e36 FLOP with 2020 algorithms.
Eventually, AI performs 100% of tasks (i.e. we train AGI) and GDP is proportional to the number of AIs we can run (which is proportional to $ on FLOP running AIs * FLOP/$ * software).

The logic described here for AI automation’s effect on GDP is the same as for its effect on R&D input. Simply replace “GDP” with “R&D work done per year” - after all GDP is simply the value of goods and services produced per year. So AI automating 50% of R&D tasks would boost annual R&D progress by >2X. (Caveat in fn.45)

The Full Takeoff Model (FTM) complicates the above simple model by assuming that a fixed fraction of tasks are performed by capital: machines and equipment. Neither AIs nor humans can perform these tasks. This means that GDP (/R&D work done per year) is never proportional to the number of AIs. In fact, even with unlimited AGIs, GDP (/R&D work done per year) cannot exceed a certain limit due to being “bottlenecked” by the amount of capital we have. I consider a few weakly-relevant sources of evidence about the strength of this bottleneck. My wild guess for these bottlenecks is that, if AI automated all cognitive tasks and we had unlimited AGIs but no additional capital, then GDP would increase by ~6X while hardware R&D progress would increase by ~100X.

For software R&D, the FTM assumes that the role of capital is instead played by physical compute for doing computational experiments. A limited amount of physical compute has the potential to bottleneck software progress. More.

The FTM has slightly lower training requirements for automating software and hardware R&D, compared with the general economy. This speeds up takeoff somewhat, because by the time AI can readily automate 20% of economic tasks it may have already automated (e.g.) 40% of R&D tasks, significantly speeding up AI progress.

What's the overall effect of AI automation on the speed crossing the effective FLOP gap? This is hard to reason about analytically, but our simulations suggest AI automation reduces the time from “AI that can readily automate 20% of tasks” to “AI that can readily automate 100% of tasks” by ~2.5X.46

45 This is the short term boost to annual R&D progress you’d get if you instantaneously automated 50% of R&D tasks. Longer term, your faster R&D progress would increase diminishing returns and so your rate of annual progress would slow over time. The short term boost calculation is roughly accurate if the effective FLOP gap is very narrow; if it’s wide then the calculation will overestimate the rate of R&D progress, perhaps significantly. This is accounted for in the FTM.

46 These results are strikingly similar to those of a simple toy model. In the toy model, crossing the effective FLOP gap corresponds to travelling along the x-axis from 1 to 0. Without AI automation your speed equals 1 unit/second throughout; you cross the entire effective FLOP gap in one second and the final fraction f of the gap in f seconds. With AI automation, your speed equals 1/x throughout, increasing over time; it turns out that you cross the entire effective FLOP gap in 0.5 seconds and the final fraction f of the gap in (f^2)/2 seconds. Time to cross final fraction f of the gap without automation / time to cross final fraction f of the gap with automation = f / [(f^2)/2] = 2/f. In our case, f = 0.8 and so the ratio is 2.5X, as found in simulation.
If you’re confused about how AI automation affects our previous calculation of takeoff speeds, I recommend looking at the version of the toy model that incorporates AI automation.

How the FTM works

The Full Takeoff Model (FTM) combines the key dynamics discussed above:

- Each timestep it calculates the effective compute in the largest training run (= software * FLOP/$ * $ on FLOP in the largest training run), then uses this to determine how many tasks AI can automate. Then it calculates how software, FLOP/$, and $ on FLOP in the largest training run increase during that timestep.

- Software and FLOP/$:
  - These increase due to software R&D and hardware R&D.
  - For the tasks not yet performed by AI, inputs of labour and capital grow as discussed in the section ignoring AI automation:
    - The human labour and physical capital invested in hardware R&D grows at ~17% after “wake up”, and at their current rate (~4%) before.
    - The human labour invested in software R&D grows at 25% after “wake up”, and at their current rate (~20%) before.
    - “Wake up” is assumed to happen when AI can readily automate sufficiently many tasks; my current best-guess value is 6% of tasks (weighted by 2020 economic value).
  - AIs perform an increasing fraction of tasks in software and hardware R&D.
    - The effects of AI automation on R&D interact multiplicatively with those from rising human investment. For example, if rising human investment doubles inputs to hardware R&D, and AI automation increases effective R&D inputs by 2X, then total hardware R&D inputs rise by 4X.
    - After “wake up” the fraction of effective compute used for software and hardware R&D grows rapidly.

- $ on FLOP in the largest training runs.
  - This increases due to a larger fraction of GDP spent on FLOP in the largest training run, and due to GDP growth.
  - The fraction of global GDP spent on FLOP in the largest training run grows as discussed in the section that ignored AI automation: initially increasing ~3X per year while we increase the fraction of chips used for training, later doubling every ~3 years.
  - AI automation increases GDP growth, which in turn increases g($ spend on FLOP in the largest training runs). For example if GDP growth increases from 3% to 7%, then g($ spend on the largest training run) increases by 4%.

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47 One caveat here: once GDP starts growing more quickly, physical capital starts growing more quickly too. This causes capital inputs to hardware R&D grow somewhat more quickly than 17%.
48 Ignoring AI automation, GDP grows at ~3%/year due to exogenously growing labour and TFP. AI automation of goods and services increases GDP growth. I don’t model AI automation of generic R&D.
Summary of core dynamics in the FTM.

A growing fraction of the world’s effective compute is used in the largest training run → AI can automate certain tasks for GDP and R&D → the world allocates growing fractions of capital, labour and effective compute to hardware R&D and software R&D tasks (the remainder is allocated to GDP) → GDP grows, hardware improves, software improves → we recalculate the stocks of capital, labour and (especially) effective compute.

In addition to this, the FTM models a number of other plausibly-important details. 49

This playground lets you see trajectories of key quantities, enter your own inputs, and see the justifications for my preferred inputs - I recommend you give it a try! Those who want a deeper understanding of how the model works should read this mathematical description of the FTM (h/t Epoch for this).

Many thanks to Epoch for coding up and running the FTM.

With my best-guess parameters, takeoff lasts ~5 years

I used my best-guess values for all parameters (discussed above and listed here): takeoff lasts ~5 years. The following graph show how the components of the largest training run evolve over time.

49 A delay from hardware R&D to producing SOTA chips (sometimes you need to upgrade fabs or construct new ones); a distinction between the cumulative stock of AI chips and the annual production (I’ve talked about the latter); a “stepping on toes” parameter (so that 10 researchers make more progress in 10 years than 100 researchers make in 1 year); assumptions about training and runtime requirements for AI that can perform x% of cognitive tasks, for all x; AIs are heavily concentrated on doing AI R&D after “wake up”; AI advantages at R&D due to e.g. faster thinking speed; ceilings on FLOP/$ and software; returns to hardware and software R&D become worse as we approach their ceilings; ceilings on the fraction of world GDP spent on hardware R&D, on software R&D and on buying FLOP; and the assumption that “wake up” happens when AI has automated 3% of cognitive tasks (weighted by their economic value in 2020).
Components of the effective compute on the largest training run over time with my best-guess parameters.

The report walks through the dynamics of this scenario and discusses other scenarios, where the parameters take conservative (slower takeoff) and aggressive (faster takeoff) values.50

---

50 “Aggressive” parameter values sometimes shorten takeoff but lead to longer AI timelines. E.g. a small effective FLOP gap has this effect (see explanation below).
This model shortens AGI timelines, compared to Bio Anchors

The Full Takeoff Model (FTM) implies shorter AGI timelines than Bio Anchors. The most important reasons are:

- **Speed-up from pre-AGI systems.** Pre-AGI systems accelerate software and hardware progress; they also increase GDP and so increase the $ spent on FLOP globally.\(^5^1\)
  - Effect size ~6 years
- **Faster growth of % world GDP spent on a training run.** Even ignoring AI automation, the FTM predicts that, a few years after “wake up”, we’ll use ~10% of global FLOP on the largest training run, with $ on FLOP globally continuing to double every ~3 years.
  - Effect size ~2 years

The FTM also models factors that make timelines longer than Bio Anchors.

The following table compares the best-guess timelines implications of FTM with those of Bio Anchors.\(^5^2\)

<table>
<thead>
<tr>
<th>FLOP to train AGI using 2020 algorithms</th>
<th>Effective FLOP gap</th>
<th>Bio-anchors timelines</th>
<th>FTM timelines</th>
<th>Timelines shift</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Year of TAI</td>
<td>Year AI could readily automate ~all cognitive labour</td>
<td>Year AI could readily automate ~all cognitive labour</td>
</tr>
<tr>
<td>~1e33</td>
<td>3</td>
<td>2043</td>
<td>2038</td>
<td>5</td>
</tr>
<tr>
<td>~1e36</td>
<td>4</td>
<td>2050</td>
<td>2044</td>
<td>6</td>
</tr>
<tr>
<td>~1e39</td>
<td>6</td>
<td>2062</td>
<td>2050</td>
<td>8</td>
</tr>
</tbody>
</table>

These timelines shifts are very sensitive to the size of the effective FLOP gap. Holding AGI training requirements fixed, a larger effective FLOP gap makes AGI sooner by lowering the training requirements for AI that significantly accelerates AI progress. More on this comparison.

\(^5^1\) Even if you don’t expect pre-AGI systems to significantly affect some sectors of GDP, it’s plausible that they significantly boost the number of AI chips produced and so of $ spent on FLOP globally.

\(^5^2\) The table compares the Bio Anchors forecast for “transformative AI” with the FTM’s forecast for “AI that could readily automate 100% of cognitive tasks”. The latter is a higher bar, so the shift in timelines is bigger than what the table suggests. The comparison assumes AGI (AI that could readily perform 100% of cognitive tasks) requires 1 OOM more effective compute to train than transformative AI.
More qualitatively, this research has increased my probability that we’ll develop AGI by 2060. Even if the training requirements for AGI are really high, the requirements for “AI that adds $trillions to GDP” or “AI that notably accelerates hardware or software progress” might be significantly lower. Hitting either of these lower bars could spur further progress that soon gets us all the way to AGI. In the language of this report, I think that avoiding AGI by 2060 probably requires both large AGI training requirements and a narrow effective FLOP gap.\textsuperscript{54}

\textsuperscript{53} [Link to diagram]

\textsuperscript{54} This is a bit oversimplified. Poor returns to hardware and software R&D could prevent us from hitting the lower thresholds in time, and hardware/software R&D progress will slow when the fraction of GDP spent on these areas stops increasing as it must do eventually. Another possibility is that AI has big economic impacts but isn’t useful for AI R&D.
Trading off training FLOP for runtime FLOP can shortens timelines

It is often possible to improve AI performance by allowing a model to “think” for longer (e.g. generating many answers, evaluating them, submitting the best); this often has the same effect as increasing training size by several OOMs.55

For example OpenAI found that {generating 100 solutions and then evaluating which is best} improved performance at solving math problems as much as increasing model size by 30X.

This suggests we could achieve the same performance as an AGI by doing a smaller training run but allowing the AI to think for longer. E.g. perhaps our training run is 10X smaller than is required for AGI, but we make up for this by giving the trained model 100X the thinking time.

Indeed, my best-guess AGI training requirements (1e36 FLOP with 2020 algorithms) and runtime requirements (1e16 FLOP/s with 2020 algorithms), with some other assumptions, imply that we will be able to run ~10 trillion AGIs by the time we train AGI. In other words, there will be an abundance of runtime compute, especially relative to the 10,000s of people working on software R&D for SOTA AI. If we can leverage this abundance we might achieve the output of, say, 1 billion AGIs long before doing a 1e36 training run.56 I think this could reduce the training run size needed for full automation by 1-3 OOMs, possibly more.

When this dynamic is included in the FTM, best guess timelines shorten by ~5 years.

More on this tradeoff.

Monte Carlo

We ran 10,000 simulations, each time randomly sampling each parameter between its “conservative” (slower takeoff) and “aggressive” (faster takeoff57) values (listed here).58

We encoded correlations between the parameters. The most important correlations are:59

55 See Jones (2021), AlphaCode, Codex, WebGPT, More.
56 For example, suppose we do a 1e34 training run and then run the resultant 1e34-AI using enough compute to run 100 billion 1e36-AIs. Perhaps, because our training run is 100X below the AGI training requirement, we get the same output as if we’d run 1 billion 1e36-AIs. We’d be trading off 2 OOMs of runtime compute for 2 OOMs of training compute.
57 Note, parameters that are “aggressive” for takeoff speed are sometimes “conservative” for AI timelines. In particular a narrow effective FLOP gap (holding AGI training requirements fixed) makes takeoff faster but delays AGI.
58 The sampling distribution is a mixture of two distributions. It places 50% weight on a log-uniform distribution between the parameter’s “conservative” and “best-guess” value, and 50% weight on a log-uniform distribution between its “best-guess” and “aggressive” values. The only exception is AGI training requirements, which are sampled from the Bio Anchors distribution.
59 These high-level correlations, and others, are recorded here; the full matrix of correlation is here. I think I’ve overestimated the correlations between these inputs, extremizing the tail outcomes. On the other hand, my sampling procedure means the parameter values never fall outside the “conservative” to “aggressive” range, which pushes in the opposite direction.
• **Strong correlation between growth of AI investments across areas.** If spending in one area of AI (e.g. software R&D) grows quickly, I also expect spending to grow quickly in other areas (e.g. hardware R&D).

• **Medium correlation between AGI training requirements and the effective FLOP gap.** If AGI requires “long horizon” training, or some other high-cost approach to training, that increases my probability that 20% of tasks will be automated with much less effective training compute than AGI.

• **Medium correlation between AGI training requirements and growth of AI investments.** If AGI training requirements are lower, it should be easier to grow AI investments quickly as they’re starting from a lower base.

We resample the parameters, except AGI training requirements, until we avoid the implication that AI can already readily automate >1% of the economy or >5% of R&D. This reduces the median sampled effective FLOP gap from 4 OOMs to 3.3 OOMs.

Here are the results, sampling AGI training requirements from the Bio Anchors best-guess distribution.61

---

60 Bio Anchors already adjusted its training requirements distribution to account for the fact that we’re seemingly not close to training TAI today; resampling training requirements here would double-count this update.

61 We assume AGI training requirements are 1 OOM higher than TAI training requirements, and reduce the probability of “you need more compute than evolution” from 10% to 4%.
### Percentile

<table>
<thead>
<tr>
<th>Percentile</th>
<th>AI timelines</th>
<th>Takeoff speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First year when AI can readily automate 100% of cognitive tasks in the general economy.</td>
<td>Years from “AI can readily automate 20% of cognitive tasks” to “AI can readily automate 100% of cognitive tasks”.</td>
</tr>
<tr>
<td>1%</td>
<td>2025.7</td>
<td>0.3</td>
</tr>
<tr>
<td>10%</td>
<td>2029.6</td>
<td>0.8</td>
</tr>
<tr>
<td>20%</td>
<td>2032.7</td>
<td>1.2</td>
</tr>
<tr>
<td>50%</td>
<td>2043.3</td>
<td>2.9</td>
</tr>
<tr>
<td>80%</td>
<td>2070.3</td>
<td>7.6</td>
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<tr>
<td>90%</td>
<td>≥ 2100</td>
<td>12.5</td>
</tr>
<tr>
<td>99%</td>
<td>≥ 2100</td>
<td>28</td>
</tr>
</tbody>
</table>

Compared to my previous best-guess scenario where AGI training requirements are $1e36$, the Monte Carlo’s median takeoff speed is faster (3 years vs 5 years). This is because:

- The median effective FLOP gap in the Monte Carlo is shorter than the best-guess (3.3 vs 4 OOMs)
- The Monte Carlo allows for the possibility that it’s easier for AI to automate cognitive tasks in {software and hardware R&D} than in the general economy, which speeds up takeoff in expectation.

---

62 This is all conditional on AGI before 2100.
Monte Carlo with aggressive training requirements

My median AGI training requirements (~1e36 FLOP using 2020 algorithms) are high compared to some. I reran the Monte Carlo on an alternative distribution with a more aggressive distribution that has a median of ~1e31.

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<td>2024.8</td>
</tr>
<tr>
<td>10%</td>
<td>2027</td>
</tr>
<tr>
<td>20%</td>
<td>2028.6</td>
</tr>
<tr>
<td>50%</td>
<td>2033.7</td>
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<tr>
<td>80%</td>
<td>2044.1</td>
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<td>90%</td>
<td>2054.9</td>
</tr>
<tr>
<td>99%</td>
<td>≥ 2100</td>
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<td></td>
<td>Years from “AI can readily automate 20% of cognitive tasks” to “AI can readily automate 100% of cognitive tasks”.</td>
</tr>
<tr>
<td></td>
<td>Tasks in the general economy.</td>
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<tr>
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<td>6.1</td>
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<td>99%</td>
<td>20</td>
</tr>
</tbody>
</table>

---

63 This is all conditional on AGI before 2100.
Unsurprisingly, lowering median training requirements by 5 OOMs makes timelines significantly shorter and takeoff significantly faster.

Alternative ways to think about takeoff speeds

Reporting the time between two somewhat-arbitrary AI capability levels gives a limited view into the dynamics of takeoff. Another approach is to ask “What does the world look like Y years before AI can readily automate all cognitive tasks?”. Here’s one example (more):

<table>
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<th>2 years</th>
<th>5 years</th>
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<td>21.1</td>
</tr>
</tbody>
</table>

More about the Monte Carlo set-up; see full results.

Parameter-importance analysis

We analysed how much varying each parameter from its “conservative” to its “aggressive” value (listed here) affected the results, holding other parameters fixed. My conclusion is that the most important parameters for takeoff speed, in order, are:

1. AGI training requirements.
2. Effective FLOP gap.
3. R&D parallelisation penalty (“If I double research efforts, how much faster is R&D progress?”)
4. Software returns.
5. How much easier is it for AI to automate the cognitive tasks in {software and hardware R&D} vs the general economy?
6. A parameter describing how much unautomated cognitive tasks bottleneck R&D progress. It influences how much AI accelerates R&D progress when it’s automated some but not all cognitive tasks.
7. How much can you reduce the training requirements for full automation by allowing AIs to think for longer? (i.e. the cap on the tradeoff between training and runtime compute.)

More on which parameters are important, and why.

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64 The median percentile for hardware will in general not be the same simulation run as the median percentile for software. And similarly for GWP and for other percentiles. The percentiles are calculated for each quantity separately.

65 The model doesn’t include lags to deploying AI, strongly suggesting that these GWP growth rates are too high.
The framework has many limitations

Here I briefly summarise key limitations of the framework, and how correcting for them would change predictions about timelines and takeoff speed.

- **Assumes no lag between developing and deploying AI.** More.
  - → impacts of pre-AGI systems happen later → AGI happens later so slower takeoff
    - But I don’t expect large lags for deployment in AI R&D and in chip production as these aren’t customer facing so face i) fewer regulations and ii) less back-lash from customers who distrust AI.
  - → bigger lag for weaker AIs than for stronger AIs → faster takeoff
    - I partly account for this by holding the necessary deployment lag fixed in the definition of the effective FLOP gap, which narrows that gap.
    - But better AI may reduce the actual deployment lag down towards the necessary deployment lag.
  - How these bullets net out depends on your exact definition of takeoff speed.
- **Assumes AI capabilities improve continuously with additional inputs.** More.
  - If progress is in fact jumpy, then there could be a fast takeoff even with a wide effective FLOP gap.
  - I find specific arguments for discontinuities unconvincing, but do assign it some probability.
- **Assumes no lag in reallocating human talent when tasks have been automated.**
  - → fewer human workers than I assume improving AI → longer timelines
    - This is important if pre-AGI systems fully automate certain jobs, less so if they partially automate jobs and workers continue to do the other parts.
- **Doesn’t model data/environment inputs to AI development.** More.
  - → takeoff could be slower if this input takes a long time to increase, or faster if it is quick to increase

---

68 I define startpoint/endpoint of the effective FLOP gap as AI that can readily perform x% of tasks, where “readily” means that the necessary deployment lag is < 1 year.

67 In particular, holding the necessary deployment lag fixed strengthens the argument “practical barriers to partially automating tasks” and so pushes towards a smaller effective FLOP gap.

69 I tentatively put ~6% on a substantial discontinuity in AI progress around the human range. (By “substantial discontinuous jump” I mean “>10 years of progress at previous rates occurred on one occasion”.) More.

70 E.g. Brynjolfsson (2018) finds that “most occupations include at least some [automatable by machine learning] tasks; (iii) few occupations are fully automatable using ML.”
○ A more comprehensive framework might define a FLOP-data gap that can be crossed with more/better data/environments or with more compute.
○ Some of the important dynamics I’ve analysed for compute (rising investment, AI automation) would apply to data as well, but others wouldn’t (the amount of high quality data isn’t already doubling every 1-3 years).

- **Doesn’t model actors’ incentives to invest in training runs and AI R&D.** Instead the model makes hacky assumptions about how investments change before and after the world “wakes up” to AI’s full economic and strategic potential. [More.](#)

More on limitations of the framework.

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### My all-things-considered probabilities

Adjusting for the above limitations, my overall probabilities change from the Monte Carlo as follows:

- About 10% more probability on <1 year takeoff.
  - Mostly from a discontinuous jump in AI capabilities allowing it to cross even a large effective FLOP (“difficulty gap”) very quickly.
- Expect somewhat slower takeoff in general (~30% longer).
  - Lags to deploying AI in AI R&D and reallocating human labour.
  - Unmodelled schlep to developing AIs (e.g. gathering data).

**My all things considered** views on takeoff speed differ from the Monte Carlo for a few of reasons, most importantly:

1. Somewhat higher probability on <1 year takeoff, due to a discontinuity in AI progress causing us to cross a medium-sized gap very quickly.
2. Longer takeoff speeds in general, due to the Full Takeoff Model ignoring various real-world frictions in developing and deploying AI.
   a. I’m not adjusting here for the possibility that we make an unusually large effort to slow down (e.g. delaying deployment by >6 months) due to caution about catastrophic risks from advanced AI. I’m just incorporating standard processes of testing and iterative deployment.

<table>
<thead>
<tr>
<th>Beliefs of the author</th>
<th>Takeoff speed</th>
<th>Percentile</th>
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<tbody>
<tr>
<td></td>
<td>Years from “AI could readily automate 20% of cognitive tasks” to “AI could readily automate 100% of cognitive tasks”. 71</td>
<td>Tasks in the general economy.</td>
</tr>
</tbody>
</table>

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71 Reminder: milestones of the form “AI could readily automate x% of tasks” require both that AI is **capable enough** to perform the tasks and that we can **run enough copies** for AI to replace every human doing those tasks. By contrast, the definition of AGI as “AI that could readily **perform** 100% of tasks” only requires that we could run one copy.
### Capabilities takeoff speed vs impact takeoff speed

Importantly, the above numbers, and my discussion more generally, has focussed on the takeoff speed of AI capabilities. To achieve the endpoint “AI could readily automate 100% of cognitive tasks” requires that AI is capable enough, and we have enough runtime compute, that AI could replace all human cognitive labour. It does not require that AI in fact has this impact.

How will AI’s impact takeoff speed differ from its capabilities takeoff speed? I think that:

- Deployment delays, e.g. due to human caution, will slow down impact takeoff speed. But competitive dynamics could limit these delays in certain strategically important fields.
- Once AI is significantly above human intelligence, it might remove these deployment delays, e.g. by disempowering humans or accelerating their deployment processes. This could mean that impact takeoff speed is faster than capabilities takeoff speed. [More](#).
  - Importantly, I expect the time from “AI actually adds $10tr/year to GDP” to “AI that could kill us if it wanted to” to be many years smaller than is predicted by the FTM, and plausibly negative, on account of deployment lags.

So overall I expect impact takeoff speed to be slower than capabilities takeoff, with the important exception that AI’s impact might mostly happen pretty suddenly after we have superhuman AI.

### What about the time from AGI to superintelligence?

So far I’ve mostly focussed on the time from AI readily automating 20% of cognitive tasks to AI readily automating 100%.

But the time from AGI (AI that can readily perform 100% of tasks, without trading off training compute and runtime compute) to superintelligent AI is also strategically important. It tells us how much time we might have to adjust to somewhat superhuman AI before there is massively superintelligent AI.

The mainline prediction of this framework is that, unless we purposefully slow down, this time period will be extremely short: probably 1 - 12 months. The reasons are:

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</table>
• **Extremely fast software progress.**
  ○ Software for AI is already doubling roughly every year, from human efforts alone. By the time we train AGI we’ll be able to run enough AGIs to increase the cognitive labour used for software R&D by >10X. What’s more, these AGIs will be super-human in some areas, not require leisure time or sleep, and potentially think much faster than humans. This suggests the first software doubling after AGI will take 1 month or less.
  ○ I argue that it’s more likely than not that there would be a ‘software-only singularity’ in this scenario, with software progress becoming faster and faster until total AI cognitive output has increased by several OOMs.
  ○ You can see this effect in the playground with the green line representing software progress going “almost vertical” as we approach AGI.
  ○ This isn’t guaranteed.
    ■ Software progress may have become much harder by the time we reach AGI.
    ■ Progress might become bottlenecked by the need to run expensive computational experiments, or to rerun multi-month long AI training runs.
    ■ If AGI training requirements are very low (<1e28 FLOP) we may not be able to run enough AGIs to significantly accelerate R&D progress.
    ■ But even with these barriers, I expect we could develop superintelligent AI within a year.

• **Fast growth of physical compute.**
  ○ Years before we have AGI, we’ll have AI that can automate a significant fraction hardware R&D, speeding up the design of new AI chips. And we’ll have AI that can increase their throughput of fabs for manufacturing AI chips, or accelerate the construction of new fabs.
  ○ This means that the quantity of physical compute in the world may be increasing very rapidly just as we first develop AGI.
  ○ You can see this effect in the playground with the yellow line representing hardware progress “rising more steeply” as we approach AGI.
  ○ Again, this is not guaranteed.
    ■ Hardware progress may have become much harder by the time we reach AGI.
    ■ Progress might be bottlenecked by the need to do physical experiments.
    ■ It’s possible that deployment lags will be long enough that we have AGI before pre-AGI systems have had significant effects on hardware improvements.

Some high-level frames for thinking about the report’s conclusions

I agree with all of these; I’ve ordered them based on how useful I find them.
• **If takeoff is fully continuous, it could be pretty fast.** An important potential source of fast takeoff is that, while AI progress is continuous, the rate of improvement is still steep enough to drive fast takeoff.\(^{72}\)
  
  a. The ‘steep rate of improvement’ is driven by:
  
  i. Not-extremely-high AGI training requirements, implying a large increase in AI capabilities *per OOM* of additional training FLOP.
  
  ii. Fast growth in the largest training run, driven by i) the fast growth of FLOP/$ and software, ii) room to significantly scale up $ spend on the largest training and incentive to do so, and iii) effects of AI automation.

• **No time for very slow takeoff unless AGI is very hard to develop.** It’s hard to maintain the following three things:
  
  a. Pre-AGI systems will have huge impacts, e.g. generating $10s-100s trillions/year.
  
  b. The training requirements for AGI are not extremely large.
  
  c. There will be decades between pre-AGI systems with huge impacts and AGI.

  These are hard to maintain because: (a) → very large increases in AI investments + significant speed-ups from AI automation → rapid increase in the largest training run. Then (b) + rapid increase in the largest training run → we train AGI within ~10 years → not-(c).

• **IEM but with quantitative predictions.** *Intelligence Explosion Microeconomics* (IEM) gave arguments for thinking AGI would lead to accelerating growth but didn’t (try to) ground things empirically or make quantitative predictions. The FTM does this via assumptions about future AI investments, the returns to hardware + software R&D, and the size of the effective FLOP gap. [More.](https://www.lesswrong.com/r/72)

• **Quantifies the tradeoff between takeoff speed and timelines.**
  
  a. Holding AGI training requirements fixed, a *slower* takeoff means AGI happens *sooner* (due to pre-AGI systems helping to develop AGI).
  
  b. The size of the effective FLOP gap controls this tradeoff. A *wider* gap makes takeoff *slower* but means AGI happens *sooner.*

• **Augments growth models to make predictions about takeoff speed.** [More.](https://www.lesswrong.com/r/72)
  
  a. Economic growth models have a strong tendency to predict that full automation of both goods production and R&D will cause large increases in both GDP and the GDP growth rate.\(^{73}\)
  
  b. According to these models, whether there is a “fast takeoff” in GDP depends on:
  
  i. How much time does it take to go from “most tasks can’t be automated” to “~all tasks can be”?\(^{73}\)
  
  ii. Once all tasks are automated, how long does it take to amass enough (computer) capital that per-task output is much higher than it was when humans were doing the tasks?
  
  c. The FTM answers (i) by assuming that: the number of tasks you can automate is tied to the largest training run you’ve done; and the threshold for automating ~all

\(^{72}\) This can be framed as a reply to Paul Christiano’s 2018 writing about takeoff speed: while he’s right that takeoff is probably continuous, it may yet be very fast. [More.](https://www.lesswrong.com/r/72)

\(^{73}\) See [section 8.2](https://www.lesswrong.com/r/72).
tasks is based on Bio Anchors; and the threshold for lower levels of automation also depends on the effective FLOP gap.
d. The FTM answers (ii) by modeling how the amount of compute and software change over time.
e. Ultimately, the FTM finds it likely that i) we probably automate most cognitive tasks (weighted by economic importance in 2020) in a 5 year period and ii) very soon after this is done, total cognitive output from AI far outstrips that from humans.

Steel man case for fast takeoff

Given all of the above, here’s my strongest case for a fast takeoff:

- **Small effective FLOP gap.**
  - Brain size – IQ correlations in humans are the only evidence we have of capability scaling in the human range, it suggests an effective FLOP gap of ~1 OOM.
  - The practical difficulties of partial automation are significant, so significant partial automation will be possible only a little before full automation.

- **Fast investment ramp up.**
  - The economic, military and strategic incentives to ramp-up investments in AI as we’re crossing the effective FLOP gap will be huge; the low current level of AI investments mean they could rise by 1-2 OOMs very quickly.

- **Other important metrics of takeoff speed will be faster than the ones I’m reporting.**
  - Time from “AI that could provide clear alignment warning shots” is probably *after* “AI that could readily 20% of tasks”, and “AI that could kill us if it’s not aligned” is probably *before* “AI that could readily 100% of tasks”.
  - The time when we actually see warning shots will be later than when we have AI that *could* provide those warning shots, making the situation even worse.
  - Impact takeoff more generally could be faster than capabilities takeoff if superhuman AI quickly removes barriers to AI deployment. More.
  - This point is very important.

- **Another framework implies takeoff will be much faster.**
  - A different one-dimensional model of takeoff implies it will take ~0.5 - 2 years.
  - However, this model ignores the fact that AI will probably surpass humans on some tasks long before others.

- **AI capabilities might not improve smoothly with additional inputs.**
  - Discontinuities *aren’t that rare*; this raises the probability of fast takeoff above the results of the Monte Carlo.

Steel man case for slow takeoff

Here’s my strongest case for a slow takeoff:
**Large effective FLOP gap, high training requirements.**
- “Short horizon” training will generate $trillions but won’t be enough to automate all cognitive tasks. That will take “long horizon training”, which will take >5 OOMs more FLOP.
- When you 300X training run size (as when going from GPT-2 to GPT-3) the performance increase isn’t *that* big, so we’ll need *many* such improvements to cross the effective FLOP gap.
- Rather than using training requirements to upper-bound the effective FLOP gap, we should use the effective FLOP gap to lower-bound training requirements.

**Slow ramp up of human investment in AI.**
- Increasing fab production is really hard as the industry is so complex.
- It will take years for new talent to add value in software and hardware R&D.
- Hardware returns are dying out fast and software progress depends on hardware progress ([more](https://example.com), [more](https://example.com)).

**Unmodelled bottlenecks will push towards slower takeoff.**
- High quality data and environments; need for new algorithmic paradigms.
- I *don’t model* delays in deploying AIs and reallocating human workers.

**AI took decades to cross human range in many narrow domains.**
- AI impacts [find](https://example.com) this in chess, Go, and checkers.
- So we should be sceptical of any framework predicting we’ll cross the human range across *most economically valuable tasks* in <10 years.
- Some possible resolutions of this tension:
  - The effective FLOP gap is on the high end of my estimates, implying high AGI training requirements.
  - Progress in those games is slower because “ML progress is faster than GOFAI progress”, there was slower investment growth, and there weren’t speed-ups from AI automation of AI R&D.
  - The range of “humans who get paid to play these games” gets crossed much more quickly than the full human range that includes amateurs; it’s the former range we’re interested in from the perspective of this report (which is about when AI could readily automate various fractions of the economy).
  - The effective FLOP gap is narrower than in those games, e.g. because “capabilities scale especially quickly in the human range” or “it’s difficult to partially automate jobs”.
- This is a very important topic for further investigation.

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**Reading recommendations after the long summary**

Here are the sections of the full report I think are most worthwhile reading after the long summary (in order).

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74 This is suggested by figure 3 of [this paper](https://example.com) and by [this graph](https://example.com).
- Evidence about the size of the effective FLOP gap - this is probably the most important and uncertain parameter for takeoff speed, perhaps after AGI training requirements.
- There's a ~65% chance of a temporary "software-only singularity", where AGIs improve software increasingly quickly while being run on a ~fixed hardware base.
- Takeoff speeds are faster according to a one-dimensional model of takeoff.
- A new version of the chimp-human-transition argument for very fast takeoff.
- Trading off training FLOP for runtime FLOP shortens timelines.
- A list of open questions.
Takeoff speeds report

Acknowledgements

- To Carl Shulman especially for introducing many of the core ideas.
- Jaime Sevilla, Eduardo Infante Roldán, and others at Epoch for coding up the Full Takeoff Model, creating the playground and many helpful suggestions.

Contents

- 0. Short summary
- 1. Long summary
- 2. What is takeoff speed? Why does it matter?
- 3. Basic framework for calculating takeoff speed
- 4. Rising AI investments
- 5. AI automation
- 6. Bottlenecks from tasks AI can’t perform
- 7. Sensitivity analysis
- 8. Limitations of the Full Takeoff Model, and how they affect takeoff speed
- Appendices
- Additional appendices

You should read the short and long summaries before reading sections in this document.

2. What is takeoff speed? Why does it matter?

I recommend skipping this section unless you’re interested to hear about ways of quantifying takeoff speeds that I didn’t end up focusing on.

This section discusses what I mean by AI takeoff speed, why it matters, and how we might quantify takeoff speed. Ultimately, I think there are multiple reasons to care about takeoff speed and multiple reasonable ways to quantify it. I introduce some ways of quantifying takeoff speed that are both decision-relevant and that I can forecast using the framework of this report.
Takeoff speed of AI capabilities need not be the same as takeoff speed of AI impacts

What question is takeoff speeds trying to answer? One vague version of the question is: How long will it take to go from significantly capable AI to billions of AGIs? 1

If it takes a month then takeoff is fast; if it takes 50 years then takeoff is slow. There are many ways you could define “significantly capable AI”; I’ll discuss some possibilities below.

If we want to avoid reference to the (arbitrary) startpoint and endpoint we can phrase the question as: How quickly will AI capabilities improve as AI systems collectively approach and surpass human intelligence?

This refers to AI capabilities directly rather than AI’s effect on the world so I call it “capabilities takeoff speed”.

Another version of the takeoff speeds question is: How long will it take to go from AI having a significant impact on the world to AI having a truly transformative impact on the world?

Again, if this happens in a month then takeoff is fast; if it takes many decades then takeoff is slow.

We could define “significant impact” in different ways depending on whether we’re interested in economic impact, unemployment, military power, technological progress, or something else. And takeoff speed will plausibly differ between these different domains based on how much AI’s effects are bottlenecked by regulations or scarce physical equipment (more). Again, in the case of economic impact, I’d define “significant” as “adding ~$5trillion/year to global GDP”.

By “transformative impact” I mean causing a transition comparable to (or more significant than) the agricultural or industrial revolution, e.g. by significantly increasing (~10X) the rate of economic growth (more).

This definition refers not just to AI capabilities but to its actual impact on the world; let’s call this impact takeoff speed.

If capabilities takeoff is fast, then impact takeoff is more likely to be fast. But they can come apart in either direction.

• Suppose there’s fast capability takeoff, but regulations, safety concerns and other bottlenecks prevent advanced AI being used in the economy. If these bottlenecks are

1 By AGI I mean an AI system, or a collection of AI systems, that can do virtually all cognitive tasks that a human can do. By “cognitive task” I mean “any part of the workflow that could in principle be done remotely or is done by the human brain”. So it includes ~all knowledge work but also many parts of jobs where you have to be physically present. E.g. for a plumber it would include “processing the visual and audio inputs relating to the problem, choosing a plan to solve it, and deciding what specific actions to take second by second”.
removed or overwhelmed gradually over many decades, you could have a slow impact takeoff.

- Suppose delays to broad adoption get shorter for more advanced AIs. Then impact takeoff will be faster than capability takeoff. An extreme case of this is where regulations stop AI have any economic impact until misaligned AGI forcibly and suddenly disempowers humanity.

I think the question of impact takeoff is probably more important than capabilities takeoff, but harder to forecast as there are more factors that influence it. The framework here is most reliable for predicting capabilities takeoff. I will report takeoff metrics that relate to both impact and capabilities, but the impact metrics don’t account for various possible delays. Before I discuss these metrics precisely, I discuss why takeoff speeds matter.

Some reasons to care about takeoff speeds

Takeoff speed is correlated with a few factors that are strategically important. For example:

- **Warning shots.** How long before the point of no return do we get clear evidence of AI risk? What about clear evidence that AI will be transformative? All things equal, faster takeoff means we’ll have less time to respond and there will be fewer people paying attention.

- **Time for high-impact alignment work.** Before we develop AIs that pose existential risk, we might develop AIs that are similar but do not pose existential risk. Alignment work on these systems will probably be particularly impactful for reducing risk, because they’ll be more similar in structure and behavior to the systems that later pose existential risk. All things equal, faster takeoff means less time to do this high-impact alignment work.

- **Concentration of power.** Faster takeoff means less time for AI progress to spread around the world. All things equal, this will lead to fewer relevant AI actors and a higher chance of an actor getting a decisive strategic advantage.

- **Changes in the strategic landscape.** Slower takeoff means more time for the world to be transformed by pre-AGI systems, e.g. dramatically changing the geopolitical landscape of the defense-offense balance in cyber. This might favour increasing longtermist influence in generic ways over making specific plans.

- **AI timelines.** Holding fixed AGI training requirements, faster takeoff means more time to AGI because earlier systems do less to accelerate AI development. This has a number of strategic implications. Indeed, this framework allows us to quantify this tradeoff.

These factors affect how large AI risk is overall and what actions we should take to reduce it.

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2 In particular, risks that could be existential as AI capabilities improve. E.g. clear evidence of misaligned power-seeking.

3 A decisive strategic advantage is “a level of technological and other advantages sufficient to enable it to achieve complete world domination”, Bostrom (2014), p. 78.
In addition, having a view about takeoff speeds might allow us to make predictions about precursors to AGI. If we’re right, we gain credibility and confidence in our views; if we’re wrong we can update our views.

**Quantifying takeoff speed**

Summary: I use GDP metrics to quantify impact takeoff speed, and consider a few different ways to quantify capabilities takeoff speed.

**Quantifying impact takeoff speed**

GDP is a useful impact metric because of its correlation with things like military power, technological progress, and the total productive capacity of civilization.

I believe sufficiently advanced AI would dramatically accelerate GDP growth. Moreover, I expect GDP growth to be steady or slow over time absent the development of sufficiently advanced AI or a small handful of other possible breakthroughs such as some forms of radical biotechnology. If GDP growth accelerates and we don’t observe other compelling causes besides AI advancement, then I think it will be reasonable to attribute the vast majority of that GDP acceleration to AI advances, and thus to use GDP growth acceleration as a measure of AI impact takeoff speed.

With this background, it is natural to identify fast takeoff with a sudden increase in GDP growth and slow takeoff with a gradual increase in GDP growth.

Paul Christiano operationalises slow takeoff as follows:

*There will be a complete 4 year interval in which world output doubles, before the first 1 year interval in which world output doubles. (Similarly, we’ll see an 8 year doubling before a 2 year doubling, etc.)*

Intuitively, this is slow takeoff because AI has a moderately transformative impact for 8 years before it begins to have a massively transformative effect.

Paul’s key metric here is the **ratio between successive GDP doubling times**. E.g. suppose GDP doubling times are as follows: 24 years (~current rate) → 8 years → 2 years → 1 year. The ratios in this example are 3, 4, and 2. Paul’s **best guess in 2017** was that the ratios would equal ~2. So I call ratios of 4 or more ‘fast takeoff’ and ratios of 2 or less ‘slow takeoff’.

Currently, my best guess is that there are a couple of ratios that are 3 or 4 during the transition, and then we settle down into ratios slightly less than 2. But it also seems plausible (>20%) that we get a ratio >8, and also plausible that all the ratios are <2.
How do these ratios compare with those observed historically in global GDP? I calculated these from David Roodman’s data set.

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The 5 doublings since 1913 all had ratios <2; the four doublings from 1100 - 1913 all had ratios between 2 and 3. AI impacts found that there was plausibly a ratio of >4 around the agricultural revolution, where my table begins.

We can also use GDP to define serial time metrics of takeoff speed. The metric I currently use is: *time from 5% GDP growth to 20% GDP growth*. 5% is significantly higher than the recent rate of 3%, so it seems like a good indicator of “crazy stuff is happening”. 20% is fast enough that I expect humans are struggling to keep up with developments.

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4 The last time annual growth for one year exceeded 5% was 2006. The last time the 5-year average for GWP growth exceeded 5% was 1974. Source.
5 We should exclude 5% growth if it’s driven by recovery from a disaster or war here. The growth should be driven primarily by frontier technological progress.
6 For serial time metrics, AI value-add to GPD is a concrete way to define a startpoint. E.g. “AI is adding $5 trillion / year to global GDP”.
Which type metric is better? Serial time metrics are easier to understand, but the startpoint and endpoint are pretty arbitrary. The ratios between successive GDP doubling times is less arbitrary, but it’s a more abstract quantity and so harder to think about.

It’s worth noting that, like any impact metric, GDP can in principle decouple strongly from AI capabilities. For example, in the growth model I ultimately use in this report, physical capital can strongly bottleneck GDP even as the amount of cognitive labour from AIs becomes extremely large. This reflects the idea that certain physical inputs are essential to (e.g.) building a house, and no amount of cognitive labour can replace them. As a result, going from 100 billion to 1 trillion AGIs might increase GDP by much less than 2X. I do expect this bottleneck dynamic to apply to some extent. But I think tracking AI capabilities explicitly when they diverge from GDP impacts is very important, so I primarily emphasise AI capability metrics (discussed below) while also reporting GDP metrics.

Another limitation of economic metrics is that they are lagging indicators of AI capabilities. Economic signs may only appear long after dangerous capabilities are developed.

What about quantifying impact takeoff speed without using GDP? You could consider other domains, e.g. military power or level of SOTA technology. I haven’t thought about how to precisely quantify takeoff speed in these domains, but the rough idea is time from “AI makes significant difference to the domain” to “AI is making abilities in this domain go through the roof”. Takeoff speed can be different in different domains.

Quantifying capability takeoff speed

Metrics of AI capabilities typically measure performance of specific systems at narrow tasks, e.g. error rate on a specific benchmark or cluster of benchmarks.

For takeoff speeds, though, I’d like a quantity that describes the collective capabilities of all AI systems across all cognitive tasks. I’m not aware of a way of quantifying this that is straightforwardly measurable. The quantities I’m using come from the growth model I use to estimate the effects of partial AI automation on R&D and GDP, and I think they will have meaningful analogues in the real world. However, they are mostly not straightforwardly measurable, at least not today. I sometimes describe these quantities as ‘metrics of takeoff’; but by this I just mean that they quantify takeoff speed, not that they’re straightforwardly measurable.

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7 More precisely, if we hold the levels of physical capital and technology fixed and increase cognitive labour to infinity, GDP only increases by ~4X in any amount of time. It only increases further once the level of technology, or amount of physical capital, increases. I discuss this further in section 6.
8 Dan Kokajlo critiques GDP metrics of takeoff speed along similar lines.
The % of cognitive tasks that AI can readily perform

In this report, I will use "AGI" to refer to an AI system, or collection of systems\(^9\), that can readily perform \(~\text{all cognitive tasks}\).

We can generalise this notion to that of \textit{AI that can readily perform }x\% \textit{of cognitive tasks}.\(^{10}\)

If AI can perform some tasks, but not others, how can we quantify the exact % of tasks it can perform? To answer this, we need some way to assign a \textit{weight} to each task that quantifies its importance. I weight each task by its economic value in 2020, as measured by the total $ that people earn while performing the task.\(^{11}\) \textbf{Throughout the report, whenever I refer to the % of cognitive tasks -- or the fraction of cognitive tasks -- I am weighting different tasks by their 2020 economic value.} I explain this concept in more detail and discuss its weaknesses in an \textit{appendix}.

With this notion at hand, we can define metrics of the form: \textit{Years from when AI can readily perform }x\% \textit{of cognitive tasks to AI that can readily perform }y\%.

This metric ignores the question: \textit{At what cost can AI perform the task?} My reason is compute is already cheap enough that we could run a human brain for \(~\$10/\text{hour}\).\(^{12}\) Compute prices will continue to fall, so I expect that once AI can perform the task it will be able to do so more cheaply than a human.\(^{13}\)

Throughout this report, whenever I say that “AI can perform” a task, I mean that it can \textit{readily} perform the task. The phrase “readily” here indicates that i) it would be profitable to do the engineering and workflow adjustments necessary for AI to perform the task in practice, and ii) these adjustments could be done within 1 year if organisations made it one of their priorities.

\(^9\) I drop the "or collection of AI systems” henceforth for brevity, even though \textit{this framework is most naturally interpreted as implying that AGI will take the form of many AIs}.

\(^{10}\) Example coarse-grained tasks include proofreading a document, writing a poem, checking a maths proof, writing code to perform a specified function, generating a strategy to meet a specified objective, giving medical advice, etc. Each of these tasks has many subtasks, which may themselves have subtasks. The tasks in this document should be thought of as the lowest level subtasks, as then we need not consider cases when AI can partially perform a task by performing some but not all its subtasks.

\(^{11}\) More precisely, the weight of task T is proportional to the total $ people earn while performing T. For each person, this is given by the time they spend on T multiplied by their hourly salary. In mathematical notation: \text{weight}_T = \text{SUM} \_{i=1}^n (\text{\$ earned performing } T \text{ by person } i) / \text{SUM} \_{i=1}^n (\text{\$ earned by person } i).

\(^{12}\) You can rent an A100 for \$1/\text{hour} and it produces \~1e14 \text{FLOP/s}. A standard median estimate of human brain FLOP/s (to the extent that that's a meaningful concept) is 1e15 FLOP/s. That implies you could run a human brain for $10/\text{hour}.

\(^{13}\) Two caveats. First, if there’s large demand for AI chips when AIs are adding $trillions to the economy, this could drive prices back up somewhat. I haven’t analysed how significant this effect could be; it will depend on how quickly chip production can be increased to meet demand. This implies that the \textit{supply of computer chips will be the key bottleneck} of how many AIs we run.

Second, it may be that the first time AI is able to perform a valuable economic task it is very expensive to run, and then the price falls over time. I discuss a model along these lines \textit{here}. In this case, it is more meaningful to track when it becomes profitable to actually \textit{automate cognitive tasks}, as opposed to when AI can first perform the tasks.
That is, if the AI could in principle perform the task if humans did a lot of work restructuring workflows and generating suitable inputs, but in practice it would take a lot of work for the AI to do this task in practice, then the AI can not readily perform the task (as I’m using the phrase). If only a relatively small amount of work is needed, however, then the AI can perform the task.  

The % of cognitive tasks that AI could fully automate

Imagine some AI can perform 50% of cognitive tasks, but there’s only enough runtime compute to run one such system. In this case, AI could not fully automate 50% of tasks because we can’t run enough AIs to replace all the human workers.

If AI can readily perform x% of tasks, and there’s enough runtime compute\(^{15}\) for AI to replace all the human workers in those tasks,\(^ {16}\) then I’ll say that AI could readily fully automate x% of tasks. (I sometimes omit “readily”, but it is always implied.)

With this notion we can define takeoff metrics of the form: Time from AI that could readily automate x% of cognitive tasks to AI that could readily automate y%.

This is a similar metric to the last subsection, but it relates not only to the capabilities of individual AIs, but to how many AIs we can run in total.

The metric I mostly focus on in the report is: Time from AI that could readily automate 20% of cognitive tasks to AI that could readily automate 100%.

How many AGIs can we run?

The Full Takeoff Model (FTM), discussed in the summary, makes assumptions or predictions about:

- When we’ll train AGI for the first time (AI that can perform 100% of cognitive tasks).
- The FLOP/s to run AGI.

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\(^{14}\) Somewhat more precisely, it should take <1 year of engineering and adjusting workflows before AI can perform the task in practice, and it should be profitable for organisations to make necessary workflow adjustments.

\(^{15}\) But at what price? As discussed above, I expect that once AI can perform the task it will be able to do so more cheaply than humans. (Though see earlier caveats in a footnote.)

\(^{16}\) For AI to replace all human workers at a task, the new AI output at the task must exceed the previous human output at that task. For example, suppose humans worldwide write 1 billion emails per day. Then for AI to replace all human workers at the task of email writing, AIs must be able to write more than 1 billion (similarly productive) emails per day. In the model of automation I use, which I explain below, once AI output rises a little above this level it becomes profitable for all human workers to work on new tasks that haven’t yet been automated (e.g. to spend all their time doing things other than emails). The numbers I report for this metric correspond to this profitability point, so are model dependent.

If previous automation has already concentrated human workers on some cognitive task, then this raises the bar for replacing all humans at that task. E.g. if automating emails causes humans to spend more time coding, then you’ll need more AIs to replace humans at coding.
• How many FLOP/s we can do at each point of time.

This means it can calculate the first year in which we have trained AGI and can run X AGIs, for any X. I like this as “year when we can run 10 billion AGIs” as an endpoint signifying when AIs’ collective cognitive abilities significantly exceed the collective cognitive abilities of humans.\textsuperscript{17} \textsuperscript{18}

Cognitive output

By “cognitive output” I mean the progress per unit time on software R&D and in other cognitive domains like maths, strategy, persuasion, etc. The restriction to cognitive domains, to the exclusion of tasks that require physical labour, captures the idea that disembodied AI will automate the stuff done by the human brain but won’t (without robotics) automate physical human labour.

My preferred unit for AI cognitive output, or for AI+human combined cognitive output, is “How many remote\textsuperscript{19} human workers would it take to add the same amount of value?” So if AIs + humans make some software progress in one month, and you’d have needed 1000 human workers to make the same amount of progress in one month without AI, then the total cognitive output of AIs + humans is “1000 remote human worker equivalents”.

Notice that in this example I looked at the total cognitive output from both humans and AIs combined. Until we have AGI, humans and AIs are complementary to each other, so it’s hard to separate out the cognitive output that’s due to “AI alone”.\textsuperscript{20} I view it as a benefit of this metric that it naturally incorporates this complementarity. Another benefit is that it avoids privileging an arbitrary capability level like ‘AGI’.

We can separate out a notion of the cognitive ‘value add’ of AI by comparing the cognitive output that would obtain if you only had human workers (with no AIs\textsuperscript{21}) with the actual cognitive output produced by the combination of humans and AIs. If the latter quantity is twice as high, we can say that AI has added twice as much value as humans alone.

\textsuperscript{17} I say “significantly” because AGI will have a number of significant cognitive advantages over humans. To list a few: run faster in serial time, smaller % of AIs in education and retirement, smaller % of time spent on leisure or sleeping, can use smaller models for easier tasks rather than doing all tasks with a fixed brain size.

\textsuperscript{18} Do AIs’ individual cognitive abilities also exceed those of humans by this point? Not necessarily. The framework sits most naturally with a comprehensive AI services interpretation of AGI, where no single AI has abilities as general as an individual human (more). But my personal expectation is that very soon in calendar time after AIs can collectively do all tasks a human can do, we’ll be able to develop a unified AI system that exceeds humans at ~all cognitive tasks. So I do think that some AIs’ individual cognitive abilities will exceed humans’ by this point.

\textsuperscript{19} Remote human workers because disembodied AIs won’t be able to do tasks involving physical labor.

\textsuperscript{20} As a concrete example, let L be the number of humans and C the number of AIs. Suppose cognitive output is given by L*C. It’s hard to attribute a fraction of this output to humans vs AIs, due to the complementarity (in this case represented via multiplication).

\textsuperscript{21} Or, more precisely, with no AIs developed after 2020. (We already use AI to help us perform cognitive tasks, and I don’t want to exclude them. I just want to exclude new AIs that automate additional cognitive tasks.)
then the AI’s cognitive value add is 2X. With this in hand, we can define the following takeoff speed metric: time from AI value add being 2X to it being 10X. This period begins when cognitive output is twice what it would be absent AI, and ends when it is 10X what it would be absent AI.\(^{22}\)

I explain how the FTM (Full Takeoff Model) calculates cognitive output here.

Impact metrics vs capability metrics

This piece will make forecasts about impact metrics and capability metrics. How much stock should we place in each?

I have greater trust in the forecasts of capability metrics. Forecasts of impact metrics involve forecasting capabilities and making substantial additional\(^{23}\) assumptions about how those capabilities translate into impact. Example assumptions:

- How much does lack of physical equipment or physical labour delay or reduce the impact of advanced AI? (We discussed this briefly above.)
- How much do regulations delay or reduce the impact of advanced AI?
- How much schlep is involved in integrating advanced AI in the economy?

These additional assumptions will tend to make forecasts of impact metrics more uncertain than forecasts of capability metrics.

In addition, the correct additional assumptions might differ in different domains. For example, perhaps lack of physical equipment will significantly bottleneck how much AGI accelerates technological progress, but won’t prevent AGI from giving its controller a huge military advantage. Or perhaps regulations will prevent AGI impacting goods and services but not software R&D. So a second advantage of capability metrics is we can make separate judgements about how AI capabilities impact multiple different domains.

On the other hand, the capability metrics are less meaningful. In particular, they will be much harder to measure and track over time, and are at some risk of involving made-up concepts derived from a growth model but not grounded in reality.

\(^{22}\) Of course, AI may automate cognitive tasks without being agentic. If AI cognitive value add is 10X, but AIs do not make plans and are not strategically aware of humans and the levers of power (see Joe Carlsmith’s draft report on AI risk), this may be much less risky than if AIs do make plans and are strategically aware.

\(^{23}\) To some extent, these additional assumptions also affect forecasts of AI capabilities. The impacts of AI on GDP and R&D accelerate future capability developments. However, I model the effect on bottlenecks on this feedback, and don’t expect large delays from regulations and schlep in back-end industries that will spur further AI development like AI R&D and chip manufacturing.
Other metrics of takeoff speed

There are a number of other serial time metrics of takeoff speed that seem plausibly useful. For example:

- Time from “AI that causes >30% of world leaders to realise that AI will be transformative” to “AI that gives its controller a decisive strategic advantage”
- Time from “misaligned AI that blatantly seeks power” to “AI that causes existential catastrophe if it’s misaligned”.
- Time from [weaker AI capability that is strategically significant] to [stronger AI capability that is strategically significant]

There will be no straightforward way to get predictions about these metrics from my framework. To do this, we’ll have to translate the AI capabilities that feature in these metrics into the language of the framework. This means mapping them to the rate of GDP growth, the % of 2020 cognitive tasks that have been automated, the number of AGIs that can be run, [the AI multiplier on cognitive output], or some other quantity that can be calculated by the model.

Summing up

We can distinguish between capabilities takeoff speed and impact takeoff speed, and have reasons to care about both. Impact takeoff speed might be more important, and it can be quantified using GDP metrics that are well grounded. Capabilities takeoff speed might be easier to predict, and it can be quantified using a few different metrics that are less well grounded. The Full Takeoff Speeds Model I’ll explain during the next few sections will make predictions about all the metrics I’ve mentioned.

3. Basic framework for calculating takeoff speed

I recommend skipping this section except that part that estimates the size of the effective FLOP gap. The rest just recaps Bio Anchors and explains the basic framework for thinking about takeoff speeds a little more slowly than in the long summary.

This section presents a very basic first-pass framework for thinking about takeoff speeds. It describes a simple extension you can make to the bio-anchors framework to get an estimate of takeoff speed.

In short, we first use the biological anchors framework to estimate the FLOP needed to train AGI. Then we add an additional assumption about the FLOP needed to train some weaker AI. Lastly, we estimate how quickly we can ramp-up training FLOP between these two points. This gives us the calendar time from the weaker AI to AGI, one metric of takeoff speed.
The rest of this section explains this basic framework in more detail. Later sections expand upon it by i) analysing how increased AI investment and incremental AI automation might affect the ramp-up of training FLOP, and ii) modelling the effect of AI automation on economic growth.

Bio anchors recap

Ajeya Cotra’s biological anchors report (hereafter, “Bio Anchors”) articulates a framework that can be used to estimate when we’ll train AGI,\(^25\) via estimating when we’ll have enough compute and software to do so.\(^26\)

In particular, it uses analogies with biological systems and trends in ML to estimate the **FLOP required to train AGI using 2020 algorithms**. That is, if AI algorithms had frozen at their 2020 levels and a multi-year concerted effort had been made to train AGI, how many FLOP would have been sufficient to succeed?

The bio-anchors report also estimates how the size of our training runs will change over time. One tricky element here is algorithmic progress: we can achieve more with each FLOP in 2025 than in 2020. To incorporate this, we can measure the size of training runs in units of **2020-FLOP**, meaning “How many FLOP would have been needed to train a system with these capabilities using 2020 algorithms?” Software progress increases the number of 2020-FLOP that are available from a fixed budget of FLOP.

The 2020-FLOP used in a training run can be calculated by multiplying together three quantities:

1. **$ on training FLOP.** How much is spent on the training run?
2. **FLOP/$.** How many FLOP does each $ buy us? This increases over time due to hardware progress.
3. **Software multiplier.** How many times more efficient are today’s algorithms than 2020 algorithms? E.g. if we could train AGI today using half as many FLOP as we’d have needed in 2020, the software multiplier equals 2.
   a. The unit for software is **2020-FLOP per FLOP**. I.e. each FLOP today corresponds to multiple 2020-FLOP because algorithms have improved.

Writing this as an equation:

\[
2020\text{-FLOP} = \text{FLOP} \times \frac{\text{FLOP}}{\$} \times 2020\text{-FLOP per FLOP}
\]

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24 Here I give a dense summary of the relevant points; readers without familiarity might want to read a summary (here or here), listen to part of this podcast, or read the full report.

25 The report actually focuses on forecasting a slightly different target: transformative AI, defined as AI which increases the rate of economic growth by \(\sim 10X\). But the same framework can be used for forecasting AGI, and this use-case will be more useful for our purposes. In what follows, I’ll talk as if the report was forecasting AGI, to simplify the exposition. The difference will matter later because AGI is plausibly harder to develop than TAI, and we’ll adjust the report’s output for this fact.

26 Specifically, Bio Anchors estimates when we’ll have enough computation to train one unified AGI system. This is aggressive because we might actually achieve AGI earlier via many distributed cheaper systems, but it’s conservative because there are inputs to developing AGI other than computation (e.g. data).
Let’s call the FLOP needed to train AGI using 2020 algorithms the **AGI training requirement** (notice that it’s in units of 2020-FLOP). When the 2020-FLOP used in a training run exceeds the AGI training requirement, bio anchors forecasts that we will train AGI.

**Extending bio anchors to estimate one metric of takeoff speed**

Bio anchors estimates the training requirements for AGI, measured in 2020-FLOP. If we add an additional assumption about the training requirements for some weaker AI system, we can estimate the *calendar time* between training the weaker system and training AGI via the growth of the 2020-FLOP used in training runs. This in turn depends on the growth of its three components: $ on FLOP, FLOP/$ and 2020-FLOP per FLOP.

**Concrete example**

Let’s go through a concrete example to illustrate this idea.

Suppose bio anchors estimates that the 2020-FLOP AGI training requirement = 1e36.\(^{27}\) I.e. it would take 1e36 FLOP to train AGI using 2020 algorithms. Then we additionally estimate that some weaker AI would take 1e30 FLOP to train using 2020 algorithms.

Then the serial time between the weaker AI and AGI is simply the time to increase the 2020-FLOP used in training runs by 6 OOMs.\(^{28}\) How long will this take? It depends on how quickly the three components of 2020-FLOP grow after we’ve trained the weaker AI. Let’s make the following assumptions:

1. **$ on training FLOP** has a growth rate of 30%.
2. **FLOP/$** has a growth rate of 40%.
3. **2020-FLOP per FLOP** has a growth rate of 40%.

The growth rate of 2020-FLOP used in the largest training run is related to the growth rates of its components as follows:

\[
g(2020\text{-FLOP}) = g(\text{$ on FLOP}) + g(\text{FLOP/$}) + g(2020\text{-FLOP per FLOP})
\]

So the 2020-FLOP used in a training run has a growth rate of 30+40+40 = 110%.\(^{29}\) This implies that it takes 13 years to increase 2020-FLOP by 6 OOMs.\(^{30}\) So we’d estimate the time from the weaker AI system to AGI as 13 years.

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\(^{27}\) Bio anchors places a probability distribution over the 2020-FLOP AGI training requirement, and I will ultimately do the same. For now though, I will proceed using point estimates to simplify the exposition.

\(^{28}\) OOM = order of magnitude

\(^{29}\) Note, this is an *instantaneous growth rate*, distinct from the *annual growth*. The former equals \(e^{gt}\); the latter equals \((1 + g)^t\). The benefit of using the former is that you can add growth rates of the components to get the growth rate of the 2020-FLOP. They’re similar when \(g < 0.1\).

\(^{30}\) \(e^{(1.1 * 12.6)} = 1\) million.
Comments on the concrete example

Firstly, the specific endpoint (AGI) and startpoint (some weaker AI) that I used here could be changed. E.g. you could use the startpoint “misaligned AI that blatantly seeks power” and the endpoint “AI that causes existential catastrophe if misaligned”.

However, the startpoints and endpoints that we can use are still fairly limited at this stage. They must both correspond to some 2020-FLOP training requirement for the methodology to work. This is plausible if they refer to some AI capability level. We can’t yet use startpoints/endpoints that refer to the number of AIs; e.g. we can’t use the endpoint “can run 10 billion AGIs”. Endpoints like this have a runtime computation requirement as well as a training requirement and we’re not yet modelling the available runtime computation. Also, we can’t yet use startpoints/endpoints that refer to GDP growth, because I haven’t introduced the constructs needed to calculate GDP. Later, with the Full Takeoff Speeds Model, we’ll have the option to use these additional startpoints and endpoints.

Secondly, it’s worth highlighting the structure of the calculation in the concrete example. It has two key inputs.

A. The effective FLOP gap between the startpoint and endpoint. In our example this was 6 OOMs. The effective FLOP gap’s precise meaning is: how many more FLOP would it take to train [endpoint AI] than [startpoint AI], using 2020 algorithms.

   a. **Note:** I sometimes just refer to it as the “FLOP gap” for short rather than the “effective FLOP gap”, but I always mean to refer to the effective FLOP gap.

B. The speed crossing the gap, $g(2020\text{-FLOP})$. In our example, 2020-FLOP had a growth rate of 110%, increasing by ~0.5 OOMs per year. The precise meaning is: what is the average growth rate of 2020-FLOP between [startpoint AI] and [endpoint AI].

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31 In the Full Takeoff Speeds Model, it turns out that the endpoint “can run 10 billion AGIs” typically comes very quickly (<2 years) after training AGI because AI automation causes hardware and software to improve extremely rapidly around this time and AGI training compute is so high that you’re not too far off being able to run 1 billion AGIs by the time you’ve trained AGI. So the endpoint “train AGI” approximates the endpoint “can run 10 billions AGIs”.
If we measure the effective FLOP gap in OOMs and measure \( g(2020\text{-FLOP}) \) in OOMs per year, we get the simple equation:

\[
\text{serial time to cross FLOP gap} = \frac{\text{FLOP gap}}{g(2020\text{-FLOP})} = \frac{\text{FLOP gap}}{g(\$ \text{ on FLOP}) + g(\text{FLOP}/\$) + g(2020\text{-FLOP per FLOP})}
\]

The rest of this section is organised as follows. First I briefly discuss considerations informing the effective FLOP gap. Then I state my bottom line about \( g(2020\text{-FLOP}) \) and compare it with the view implicit in bio-anchors.

**Note**, the report’s main metric “time from AI that could readily automate 20% of cognitive tasks to AI that could readily automate 100%” implicitly makes reference both to whether AI could perform the tasks and to whether we can run enough AIs to replace humans at the tasks. So at this stage I can only calculate this metric by i) defining “weaker AI” as AI that can readily perform 20% of tasks, and ii) assuming that there will be enough runtime compute to automate tasks once AI can readily perform them (which is not true when AGI training requirements are low).

Evidence about the size of the effective FLOP gap

The choice of effective FLOP gap presupposes some startpoint and some endpoint. For concreteness I’ll use startpoint = AI that can perform 20% of cognitive tasks, endpoint = AGI (AI that can perform ~100% of cognitive tasks). Crossing the effective FLOP gap involves going most of the way in capability space from today’s AI to AGI. This input to takeoff speeds is second only to AGI training requirements in terms of being very important but very uncertain.
I think Hans Moravec’s "rising tide of AI capacity” visualisation is useful for framing this discussion (h/t David Schneider-Joseph for this point). Currently AI can only do a small fraction of cognitive tasks – the areas of the map that are currently underwater. Over time the AI capabilities improve (the tide rises) and AI can perform more and more tasks. Eventually, AI can readily perform all cognitive tasks (everything is under water).

Hans Moravec’s "rising tide of AI capacity” can help us think about the meaning of the effective FLOP gap

For our purposes, we should imagine the surface area of the landscape to be proportional to the tasks’ economic value in 2020. (Or, if we’re thinking about R&D automation, proportional to the task’s share of R&D.) Then the effective FLOP gap tells us how much more effective training compute we need to cover all the landscape compared to just 20% of it.

There are a few factors that can weakly inform the choice of effective FLOP gap:

- **AGI training requirements** bound it from above.
- **SOTA AI capabilities** weakly bound it from below.
- **Horizon length** suggests it could be pretty big.
- **How AI capabilities vary with training FLOP between different domains** provides an estimate.
- **How AI capabilities vary with training FLOP within a domain** provides a low-end estimate.
- **How animal capabilities vary with brain size** provides a low-end estimate.
- **How human capabilities vary with brain size** provides a low-end estimate.
- **Practical barriers to partially automating tasks** suggest it could be very small.

I’ll discuss each factor in turn.
AGI training requirements. As of today the largest training run is \(\sim 3 \times 10^{24}\) FLOP.\(^{32}\) My median guess is that AGI training requirements are \(1 \times 10^{36}\) 2020-FLOP,\(^{33}\) so my median effective FLOP gap can be no bigger than 12 OOMs. More generally, shorter AI timelines lead to a smaller effective FLOP gap.

SOTA AI capabilities. In my opinion, today’s AI systems are not close to being able to readily perform 20% of all cognitive tasks done by human workers. (Actually automating these tasks would add \(\sim 10\)\(\times\)10\(^{12}\) to GDP.\(^{34}\))

a. Based on this, and my rough sense for how much progress we’re getting with each additional OOM (informed by looking at scaling papers and playing around with GPT-2 and GPT-3), I’d want to put my startpoint above \(1 \times 10^{27}\) 2020-FLOP.

b. Another rough-and-ready approach is to naively extrapolate recent trends in AI value-add and model size. This suggests the startpoint should be \(>3 \times 10^{28}\) 2020-FLOP.

- The data I could find suggests AI value-add is doubling roughly every 2 years,\(^{35}\) over which time training runs have increased by \(\sim 1\) OOM.\(^{36}\) The largest training run as of July 2022 is \(3 \times 10^{24}\). If today’s systems could readily add \$500\(\times\)10\(^{12}\) per year to the economy, that would correspond to automating \(-1\)% of cognitive tasks.\(^{37}\) If each doubling of value-add continues to take \(\sim 1\) OOM, AI won’t automate 20% until \(>3 \times 10^{28}\) FLOP.\(^{38}\)

c. Overall, I’d want to put my startpoint above \(1 \times 10^{27}\) 2020-FLOP and probably above \(1 \times 10^{28}\) 2020-FLOP. But if I condition on AGI requiring (say) \(1 \times 10^{30}\) 2020-FLOP then I’d want to make it lower.

Horizon length.\(^{39}\)

a. Suppose we automate some economic tasks using a horizon length of 1 second but training AGI requires a horizon length of 1 year with the same model size or bigger. This implies a effective FLOP gap of \(>7.5\) OOMs.\(^{40}\)

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\(^{32}\) I believe these were the requirements for PaLM.

\(^{33}\) The Bio Anchors best-guess median training requirement for TAI is \(1 \times 10^{35}\); I add 1 OOM to account for AGI being harder than TAI.

\(^{34}\) World GDP is \(\sim \$100\)\(\times\)10\(^{12}\), about half of which is paid to human labour. If AI automates 20% of that work, that’s worth \(\sim \$10\)\(\times\)10\(^{12}\)/year. [This is a bit aggressive, as many tasks have a component of physical labour (though all have some cognitive component). On the other hand, AI will probably produce more output at those tasks than the humans they replace (as they’re cheaper to run), increasing their value-add.]

\(^{35}\) E.g. here, here, here, here. I don’t know how reliable these estimates are, or understand their methodologies.

\(^{36}\) Epoch's piece.

\(^{37}\) World GDP is \(\sim \$100\)\(\times\)10\(^{12}\), about half of which is paid to human labour. If AI automates 1% of that work, that’s worth \(\sim \$500\)\(\times\)10\(^{12}\)/year.

\(^{38}\) The bound would be higher if I used a number below \$500\(\times\)10\(^{12}\), or if I included software progress. OTOH, value-add per OOM of training FLOP may rise.

\(^{39}\) This concept is from bio anchors. Ajeya defines it as follows: How much data the model must process (on average) to tell with a given level of confidence whether a perturbation to the model improves performance or worsens performance.

\(^{40}\) There are 30 million seconds in a year.
b. My median guess would be that, compared to this, the startpoint will require longer horizons and we will train AGI with shorter horizons. So my extremely tentative takeaway would be that the effective FLOP gap is $>5$ OOMs.\(^{41}\)

c. I put (even) less weight on this consideration than the others because the usefulness of the 'horizon length' concept is debatable and I don’t know whether the startpoint-AI will use a very different horizon length to AGI.

d. If I condition on very large training requirements for AGI, however, I become more convinced by this consideration. At that point I think AGI requires very long horizons, but still think that short horizons will produce significant economic value.

- **How AI capabilities vary with training FLOP between different domains.** How much do training FLOP requirements vary across different tasks or domains?
  
  a. **AI has strong comparative advantages in some domains relative to others.** Probably, less training FLOP will be needed for automating human work in domains like these. Indeed, this is how AI can *already* perform at super-human level in some domains. There are many mechanisms that can give AI comparative advantages at some tasks but not others:
    - Some tasks can be performed by AI with “short horizon training”, others require “long horizon training”.\(^{42}\)
    - Some tasks require strong sim2real transfer.
    - Some tasks are more similar to tasks used in AI pre-training.
    - Some tasks have much more available data from demonstrations or human feedback.
    - Some tasks benefit from memorising lots of information.
    - For some tasks it’s important to “always be on” (no sleep), or to consistently maintain focus (no getting bored or slacking).
    - For some tasks, it’s much easier to verify that an answer is correct than to generate the correct answer.
    - People choose to develop specialized AI architectures and training processes for some tasks but not others.
    - Some tasks require very high levels of reliability (e.g driving vs drafting an email).

  b. **GPT-N task performance.**
    - Lukas Finnveden has extrapolated the performance of GPT-N on a variety of benchmarks. You could look at the graphed extrapolations to compare the (predicted) training FLOP needed to solve different benchmarks. In the linear extrapolation, the first benchmark exceeds a score of 90% $\sim4$ OOMs before the last benchmark. In the sigmoid extrapolation, the gap is $\sim5$ OOMs. So we’re on $\sim4$-5 OOMs.

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\(^{41}\) There are $\sim5$ OOMs between a “10 second” horizon length and a “1 month” horizon length.

\(^{42}\) This concept is from [Bio Anchors](https://example.com). Short horizons means that the model only needs to “think” for a few seconds for each data point; long horizons means the model needs to “think” for months for each data point and so training requires much more compute.
Both extrapolations omit the task with the worst SOTA performance (ANLI).

On the other hand, they both include the text-completion task, which is very similar to GPT’s training task. Omitting this lowers the estimated effective FLOP gap to ~4 OOMs.

As before, this scaling would be smaller using Chinchilla scaling laws, and perhaps smaller still the scaling law we’re on when we start crossing the effective FLOP gap. So I’ll reduce that gap to ~3 OOMs.

How does the gap for the extrapolated benchmarks compare to the gap we might see for economically valuable tasks? I expect the latter gap to be bigger. These benchmarks were selected (in part) for being appropriately challenging to SOTA LMs, which will narrow the range of difficulty between them. Economically useful tasks will not have this selection pressure, and are generally much more varied in general (e.g. they’re not all language based). I’ll tentatively add another 2 OOMs for this.

So overall I take this consideration to suggest that the effective FLOP gap is >3 OOMs and use a tentative best-guess of ~5 OOMs.

c. RL vs transformer training FLOP

- AlphaStar had 139 million parameters and took ~2e23 FLOP to train. GPT-1 had a similar number of parameters (117 million) but only took 1e19 FLOP to train. That’s a difference of 4 OOMs.
  - The difference is probably partially due to AlphaStar having a longer horizon length, and I discussed horizon length above. But other factors contribute, like the use of league-based training and training data being more noisy.

I think this is representative of a broader pattern of RL systems having several OOMs more training FLOP than similarly-sized LMs (where size is measured in parameter count, or in FLOP per forward pass).

- [Someone could check this by looking at Xland.]

If some economically valuable tasks will be automated by transformer architectures of a certain size but other tasks are only automated by RL systems of a similar size, then I’d expect the effective FLOP gap to be >4 OOMs.

This would combine with other sources of a wide effective FLOP gap.

d. To sum up:

- AI comparative advantages: suggests that the gap is wide in general.
- GPT-N: >3 OOMs, ~5 OOMs.
- RL vs transformer: maybe implies an additional ~4 OOMs.

e. Overall, I interpret this ‘training FLOP differences in different domains’ consideration as suggesting that the effective FLOP gap is >3 OOMs, weakly

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43 See page 3 here.
44 From eyeballing figure 3 of Compute Trends Across Three Eras of Machine Learning.
suggesting a best guess of \(~5\) OOMs, and lending plausibility to amounts as high as \(~8\) OOMs.

- **How AI capabilities vary with training FLOP within one domain.** When you increase training 2020-FLOP within one domain, how much do capabilities improve? What increase in training FLOP is needed to cross the human range?
  
  a. **GPT-N.** I believe GPT-3 required about \(~2.5\) OOMs more training 2020-FLOP than GPT-2.\(^\text{45}\) People familiar with both systems can use the difference to intuit how much performance improves with more training FLOP. Intuitively, the difference is pretty big! My extremely rough sense is that once AI can readily perform 20% of language based tasks, you’d need \(~two\) similar-sized improvements before it could readily perform all language based tasks,\(^\text{46}\) suggesting an effective FLOP gap of \(~5\) OOMs.
    - The difference in training FLOP between GPT-2 and GPT-3 would be smaller with the new Chinchilla scaling law, and by the time we’re crossing the effective FLOP gap we may be using better scaling law still. (For example, certain prompting or fine-tuning or aggregation techniques may improve how much total performance increases with scale, as suggested by [figure 3 here].) So I’ll put my overall estimate here at \(~4\) OOMs.
  
  b. Other papers on "scaling laws" could potentially be similarly informative here.
    - The LM-scaling papers I’ve glanced at, seem broadly consistent with the above, with \(~2\) OOMs of training FLOP improving the score on large aggregations of benchmarks (e.g. BIG-Bench, MMLU) by \(~10\)-\(~40\)%.
  
  c. **Go.** The difference between an intermediate amateur Go player and the best in the world is \(~2800\) Elo.\(^\text{47}\) Marginal doublings of training FLOP improved AlphaGo’s Elo by \(~300\) - \(~700\).\(^\text{48}\) This implies that \(~4\) - \(~9\) doublings of training FLOP are needed to cross the human range, or \(~1\) - \(~3\) OOMs. I put more weight on this than the GPT comparison, as the comparison to human abilities is more grounded.
    - Though given that we’re talking about the effective FLOP gap for *economic value*, we should make the startpoint when you can get paid for your performance. If we measured the difference from “a bit below professional” to “best in the world”, the Elo gap would be less than half as big.\(^\text{49}\) Then the FLOP to cross the range would be \(~0.5\)-\(~1.5\) OOMs.

  d. So to sum up:

\(^{45}\) This data suggests GPT-3 took \(~100\)X more training FLOP; I’m assuming \(~3\)X algorithmic improvements on top of that.

\(^{46}\) Obviously, comparing the qualitative capability gap between GPT systems to the gap between top-performing and low-performing humans is fraught. One worry is that we’re less able to notice intelligence differences between systems much dumber than us, compared to systems similarly intelligent to us.

\(^{47}\) See this sheet.

\(^{48}\) From figures 3, 4 and 5 of the AGZ paper. See reasoning on this sheet.

\(^{49}\) The world’s best Elo is 3800. Go’s “professional” range starts at 2700 Elo, while and Elo corresponds to “advanced amateur”. Using 2400 Elo as the start point would give us a “human range” of 1400 Elo.
- GPT-N scaling: ~4 OOMs
- Go: ~1 OOM

e. These results underestimate the effective FLOP gap because crossing the human range in one domain is easier than crossing the range across all economically valuable tasks. I.e. they ignore the above point of AI having comparative advantages at some domains over others.
   - This is especially true for Go, which is just one game.
   - It’s only somewhat true for GPT-N, as the full space of cognitive tasks isn’t that much broader than language-based tasks.

f. So overall I interpret this as suggesting >4 OOMs as a soft lower bound, and ~5 OOMs as a very tentative best guess.

- How animal capabilities vary with brain size.
  a. One comparison here is humans vs chimps. Human brains are probably ~3X bigger than chimp brains (in terms of FLOP/s), and there is arguably a large gap in cognitive abilities. Perhaps humans have an additional ~3X from software improvements. If you think “chimp → human” is enough to cross the effective FLOP gap, this implies a effective FLOP gap of ~1 - 2 OOMs.
  b. Rat brains are 2 OOMs smaller than human brains (in terms of synapses and so FLOP/s). Perhaps accounting for software, the difference is 3 OOMs. It’s not crazy to me that rats could perform 20% of cognitive tasks if they’d been selected to do so. This suggests an effective FLOP gap of ~4-6 OOMs.
  c. I find the chimp comparison more plausible.
  d. Both approaches ignore AI having comparative advantages at some tasks over others, so underestimate the gap.
  e. Overall, I see this as lending weight to effective FLOP gaps as low as 1 OOM, and weakly suggesting a best-guess of ~3 OOMs.

- Brain size - IQ correlations within humans.
  a. There’s a few decent-seeming papers that estimate the correlation between brain volume and IQ. My conclusion from a few hours looking at these was that a 10% increase in brain volume might cause a gain of ~4.5 IQ points. More.

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50 Chimps have about 3X fewer neurons than humans, and the data in figure 4C of this paper suggests they have a little over 3X fewer synapses. (Synapses are closer to what we care about for estimating FLOP/s.)

51 Two methods: Method 1 anchors to runtime FLOP/s, method 2 anchors to total lifetime learning FLOP.
   Method 1: Human runtime FLOP/s is ~10X bigger (including software gains), implying 100X more training FLOP with Chinchilla scaling.
   Method 2: Human lifetime learning uses ~3X more FLOP than chimps as the learn for about the same length of time. Bump this up to 10X for better software for learning. Then assume training FLOP will be proportional to lifetime learning FLOP. Method 2 assumes ML training-run scaling will be as good as hominid life-learning scaling. Hominid scaling might be better because the life-learning algorithm is better than our training algorithms; it might be worse because it doesn’t scale data optimally with brain size but we will be able to do this in ML.

52 Two methods: Method 1 anchors to runtime FLOP/s, method 2 anchors to total lifetime learning FLOP.
   Method 1: 3 OOMs bigger brain → 3 OOMs more runtime FLOP/s → 6 OOMs more training FLOP with Chinchilla scaling.
   Method 2: 3 OOMs bigger brain and 1.5 OOM longer childhood → 4.5 OOMs more training FLOP.

53 I’ve largely taken this line of reasoning from this doc by Paul Christiano.
b. If we assume that brain volume is proportional to brain FLOP/s, then a 10% increase in brain FLOP/s causes a gain of 4.5 IQ points.
   - Between primates, it seems like # neurons and # synapses are roughly proportional to brain volume, confirming this assumption.\textsuperscript{54}
   - It seems that within humans, brain volume is more slightly anti-correlated with neuron density, implying that a 10% increase in brain FLOP/s could have a larger effect on IQ.\textsuperscript{55} For now, I'll err conservative and leave this out.

c. In line with the spirit of Bio Anchors, I'll assume that a 10% increase in AI model size (measured in FLOP/s) has the same impact on IQ as a 10% increase in human FLOP/s.\textsuperscript{56} [I'll later consider a more aggressive assumption.]

d. So 10% bigger AI model size \(\rightarrow\) \(~4.5\) IQ points.

e. So a 10X bigger AI model \(\rightarrow\) \(~24\) 10% increases in model size\textsuperscript{57} \(\rightarrow\) \(~110\) IQ points.

f. Let's assume training FLOP increases with the square of model size.\textsuperscript{58} Then 100X more training FLOP \(\rightarrow\) 10X bigger model \(\rightarrow\) \(~24\) 10% increases in model size\textsuperscript{59} \(\rightarrow\) \(~110\) IQ points.

g. Intuitively, going IQ 45 \(\rightarrow\) IQ 155 would cross the effective FLOP gap (initially able to perform <20% of economic tasks \(\rightarrow\) then able to perform \(~100\)%). So this naively suggests an effective FLOP gap of 2 OOMs.

h. You could plausibly end up with a smaller effective FLOP gap:

\textsuperscript{54} I expect # synapses to be proportional to FLOP/s. \textit{Figure 4A} of \textit{this paper} finds the # synapses per unit volume is constant. If neurons matter, \textit{this paper} claims the # neurons is slightly less than proportional to volume (# neurons = volume\(^0.9\)). (Would be interesting to check if these two claims are consistent with the slight increase in synapses per neuron observed in figure 4C of the first paper.)
\textit{This paper} claims the number of neurons is proportional to brain volume and
\textsuperscript{55} Pakkenberg & Gundersen 1997 (\(N=94\)) is the only thing I'm aware of studying correlations between brain volume and neuron or synapse count in humans (thanks to Tegan for sharing). Bottom line: I think the data naively imply a 10% increase in FLOP/s would add 5.8 IQ points. I may have made a math mistake though.

Figure 4 shows that brain volume is anti-correlated with neuron density (# neurons per unit volume), so that an 85% increase in brain volume is only associated with a 61% increase in neuron count. This implies that each 10% increase in brain volume increases neuron count by 7.7%, with humans. (I.e # neurons = volume\(^0.77\).) Assuming FLOP/s per neuron is constant, each 10% increase in brain volume increases FLOP/s by 7.7%. So a 10% increase in FLOP/s would be equivalent to a 13% volume increase and increase IQ by \(~5.8\) IQ points. (4.5\(\times\)1.3=5.8.)

We can sense-check this using the raw correlation between neuron count and brain volume reported in table 3. The correlation is 0.71. This is broadly consistent with 0.77 from above, as neuron count had larger variance than brain volume. (E.g. If 1 SD of brain volume was 5% of the mean brain volume, then 1 SD of neuron count was >5% of the mean neuron count.)

\textsuperscript{56} An increase in AI model size might be better than human brain size due to the ability to do more calculations in series, or because we'll make small and easy adjustments to AI algorithms to make them suited to the new scale.

\textsuperscript{57} \(\ln(10)/\ln(1.1) = 24.2\)

\textsuperscript{58} I.e. Chinchilla scaling. The basic rationale here is double the FLOP/s \(\rightarrow\) double the # parameters \(\rightarrow\) double the # data points needed for training. Also double the FLOP/s \(\rightarrow\) double the FLOP per data point. With 2X the data points and 2X FLOP per data point, training FLOP increases 4X.

\textsuperscript{59} \(\ln(10)/\ln(1.1) = 24.2\)
Assume training compute is proportional to lifetime learning compute. Humans with bigger brains don’t get more “data” (experience) to learn from, so a 10X bigger human brain would use only 10X as much compute to learn. Anchoring to this, we might need to only use 10X as much training compute, an effective FLOP gap of 1 OOM. ⁶⁰

Use a smaller IQ gap. If we used IQ 70 → IQ 125 then the effective FLOP gap would halve to 1 OOM.

Doing both of the above reduces the effective FLOP gap to 0.5 OOMs!

i. Overall, I take the estimate here to be ~1 OOM.

j. Again, a very significant counterpoint is that early AIs will have strong comparative advantages at some tasks over others. E.g. someone with IQ 45 can’t multiply 7 digits numbers in their heads, and yet we have calculators.

k. Overall, I see this as lending some plausibility to gaps as low as 0.5 OOMs as well as weakly suggesting a best-guess of ~2-3 OOMs.

Practical barriers to partially automating tasks.

a. A high-level task like “present on topic X” might have many subtasks like “learn about X”, “plan the presentation”, “write the presentation”, “check for errors”, “deliver the presentation”. These subtasks themselves have many further subtasks (“search for relevant articles”, “extract relevant information from articles”), and so on.

b. Before AGI, a lot of AI’s economic impact will probably come from partially automating high-level tasks; i.e. from automating some subtasks but not others. ⁶¹

c. But partial automation may be difficult in practice, e.g. if it involves integrating AIs in complex workflows. For example, suppose an AI can write a presentation from a suitably formatted plan. Using this AI to partially automate “present on topic X” would require somehow creating a plan in a format suitable for the AI.

d. This could mean that, in practice, it is only when AI is close to being able to fully automate a high-level task that it does significant amounts of partial automation.

e. In addition, it may be the case that AI is able to fully automate most high-level tasks at about the same time because they require similar capabilities (or similar subtasks).

f. Combining the above two bullets, we may automate most high-level tasks at about the same time and only achieve significant partial automation of high-level tasks shortly before full automation. ⁶² This implies there might only be a short gap

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⁶⁰ This implicitly assumes human lifetime-learning scaling via increasing brain size is as good as ML scaling by increasing training FLOP. Lifetime learning might be better because the life-learning algorithm scales better than ML training algorithms; it might be worse because ML algorithms can scale the amount of data optimally with training FLOP, unlike lifetime-learning.

⁶¹ AI may also allow us to restructure the high-level task entirely so the necessary subtasks change, and automate some of the new subtasks. It may also allow us to do new kinds of high level tasks.

⁶² One concrete way to think about this is as follows. Suppose there are 100 tasks on the lowest level. Each high-level task requires 90 of them, so there’s lots of overlap between high level tasks. But it’s only possible to partially automate a high-level task once 80 out of its 90 lower level tasks can be automated, due to practical difficulties with partial automation. Now imagine low-level tasks are automated one by one in random order. In this toy model, all the high-level tasks will be automated at a similar time (when we’re
from “begin to significantly automate some high-level tasks” to “fully automate ~all high level tasks”.
g. This all pushes towards a smaller effective FLOP gap, somewhat smaller and perhaps significantly smaller than we’d have otherwise thought.
h. This effect is **significantly increased** by the fact that my definition of “AI can readily perform task X” is “it would be profitable for organisations to do the engineering and workflow adjustments necessary for AI to perform task X in practice and they could make these adjustments within 1 year if they prioritised them”.
   - If it takes many decades to cross the effective FLOP gap, there would be much more time to rearrange workflows to allow for partial automation. However, the numbers I’m getting out of the full takeoff speeds framework suggest we’d cross even a large effective FLOP gap in <20 years.

This table summarises my very tentative takeaways from each factor.

<table>
<thead>
<tr>
<th>Factor</th>
<th>How it informs the effective FLOP gap</th>
<th>My tentative takeaway for the effective FLOP gap</th>
<th>How much relative weight I place on each consideration (1-5)</th>
</tr>
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<tbody>
<tr>
<td>AGI training requirements</td>
<td>Constrains the endpoint</td>
<td>Low AGI training requirements bound the effective FLOP gap from above.</td>
<td>5</td>
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<tr>
<td>SOTA AI capabilities</td>
<td>Constrains the startpoint</td>
<td></td>
<td>3</td>
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<tr>
<td>Horizon length</td>
<td>Directly informs choice of effective FLOP gap</td>
<td>&gt;5 OOMs</td>
<td>2 (more if training requirements are large)</td>
</tr>
<tr>
<td>How AI capabilities vary with training FLOP between different domains</td>
<td>Directly informs choice of effective FLOP gap.</td>
<td>&gt;3 OOMs</td>
<td>4</td>
</tr>
<tr>
<td>How AI capabilities vary with training FLOP within a domain</td>
<td>Directly informs choice of effective FLOP gap.</td>
<td>&gt;4 OOMs</td>
<td>3</td>
</tr>
<tr>
<td>How animal capabilities vary with brain size</td>
<td>Directly informs choice of effective FLOP gap</td>
<td>~1 OOM is plausible</td>
<td>2</td>
</tr>
<tr>
<td>How human capabilities vary with</td>
<td>Directly informs choice of effective FLOP gap</td>
<td>~0.5 OOM is plausible</td>
<td>3</td>
</tr>
</tbody>
</table>

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close automating all 100 low-level tasks) and each partial automation of a high-level task only happens shortly before full automation.
Overall, before taking into account AGI training requirements my best guess for the effective FLOP gap is ~4 OOMs, but I wouldn’t be surprised by 1 OOMs or 8 OOMs.

If I condition on low AGI training requirements of 1e30 2020-FLOP the first two factors bite hard and my best guess is ~2-3 OOMs; if I condition on large AGI training requirements of >=1e38 2020-FLOP the “horizon length” factor and “How AI capabilities vary with training FLOP between different domains” comes into play more and my best guess is ~5-6.

What bio-anchors says about speed crossing effective FLOP gap

The effective FLOP gap is measured in 2020-FLOP. The bio anchors report projects its three components over time. In Ajeya’s best guess sheet their growth rates are as follows:

1. $ on FLOP for a training run initially doubles every 2.5 years (growth rate 28%) until it reaches 1% of US GDP. Then its growth rate is 3%.
2. FLOP/$ doubles every 2.5 years (growth rate 28%) until it reaches a maximum. The growth rate slows somewhat when the maximum is near.
3. 2020-FLOP per FLOP doubles every 2-3 years (growth rate ~28%) until it reaches a maximum.

This implies that g(2020-FLOP in the largest training run) is initially ~84% and then slows down to ~59%. This corresponds to 0.36 OOMs per year initially followed by 0.26 OOMs per year. At the faster pace, it would take 11 years to cross an effective FLOP gap of 4 OOMs; at the slower pace it would take 15 years.

Summing up and looking ahead

I’ve introduced a first-pass framework for calculating some metrics of takeoff speed. At the moment, it can calculate the calendar time between training some weaker AI and some stronger AI; it cannot calculate metrics relating to the number of AIs or their effects on GDP.

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<table>
<thead>
<tr>
<th>brain size</th>
<th>FLOP gap</th>
<th>Effective FLOP gap smaller</th>
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63 One thing informing this is calculating a weighted average using an adjusted version of the above table.
64 The growth rate slows somewhat when the maximum is near.
65 The precise doubling time depends on the biological anchor. For medium and long horizon anchors it is 2 years, for short horizon anchors it is 3 years.
66 28 * 3 = 84
67 3 + 28 + 28 = 59
68 4/0.36 = 11
69 4/0.26 = 15
In this framework, the takeoff speed depends on the size of the effective FLOP gap and our speed crossing it. Our speed crossing it is given by the equation \( g(2020-\text{FLOP}) = g(\$ \text{ on FLOP}) + g(\text{FLOP}/\$) + g(2020-\text{FLOP per FLOP}) \).

The next few sections extend this basic model. Section 4 estimates \( g(2020-\text{FLOP}) \) in the run-up to AGI, based on the effect of fast rising AI investments. Section 5 models the effect of incremental AI automation in the run-up to AGI. The infrastructure introduced here allows us to calculate metrics based on the number of AIs and on their effects on GDP. Section 6 discusses bottlenecks. Later sections perform sensitivity analyses, discuss its many limitations, and come to an all-things-considered bottom line.

4. Rising AI investments

*I would treat this section as providing detailed parameter estimates for important inputs to the Full Takeoff Model. In particular, it estimates the pace at which human investments on the largest AI training run, hardware R&D and software R&D will grow. It also estimates the returns to hardware and software R&D. It then infers how fast takeoff will be from human investment alone, essentially calibrating this toy model.*

Summary

*I believe* pre-AGI systems have the potential to increase GDP by $10s trillions and probably $100s trillions *per year*. Given this, and the fact that AI investments are currently in the $10s billions (sources below), I expect human investments in AI to grow very rapidly after key actors “wake up” to the potential economic and strategic\(^{70} \) value of AI. While governments and chip manufacturers and investors are aware that AI is strategically important, I claim their actions implicitly significantly underestimate its transformative potential.

This section analyses the effect of this fast-rising investment on the three components of 2020-FLOP.

The numbers in this section are best characterised as “wild guesses informed by weakly relevant empirical data”. As such, the uncertainties are very high. That said, here are my tentative central estimates.\(^{71} \) After “wake up” I guess that:

- $ on FLOP in the largest training run will initially grow at \( \sim 97\% \), and then later at \( \sim 22\% \)
  - $ on FLOP globally will grow at \( \sim 22\% \), based on how quickly I guess we’ll be able to expand chip production after “wake up”.

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\(^{70}\) e.g. via military and security applications.

\(^{71}\) For now I’m leaving these all at 2 significant figures so I don’t lose information about the centres of my subjective distributions, but 1 significant figure would be much more appropriate given the magnitude of the uncertainties involved.
% global FLOP on the largest training run will grow at ~75%, and have 1 - 3
OOMs room to grow in total.

- FLOP/$ will eventually grow at up to ~88% (recently ~25%), driven by real $ hardware
  investment growing at ~17% (recently 5%). But it will take many years before it grows
  this quickly, gradually accelerating from its recent pace of doubling every ~3 years.
- 2020-FLOP per FLOP will grow at ~31%, driven by real $ software investment growing
  at ~25% (recently ~20%).

This section often refers to growth rates. If you prefer thinking in terms of doubling times, use
this converter.

Background - “waking up” to advanced AI’s economic potential

I believe AGI would drive explosive economic growth - Gross World Product (GWP) growing at
>= 30%/year. I think less powerful AI could drive multiple doublings of GWP. Nominal GWP is
$85tr, and so doubling GWP even once involves adding ~$80tr to the global economy (starting
from now). In other words, on my view, pre-AGI systems have the potential to generate many
$10s or $100s of trillions per year.

Annual investments into hardware and software for AI development are 2-4 OOMs smaller:

- All hardware. Total semiconductor industry revenues are $550 billion; semiconductor
  R&D is ~$70 billion and semiconductor capex was ~$130 billion.
- AI hardware. The size of the AI chip market is probably ~$20 billion;\(^2\) AI targeted chip
  R&D is maybe a few billion $\(^3\).
- AI software. I’d guess annual spending on software workers for SOTA AI is $10-100
  billion\(^4\).

I haven’t dug into these numbers and this is important further work.

I believe that before AGI is developed, many key actors\(^5\) will “wake up” to the potential for
advanced AI to generate $10s trillions per year. At the very latest, this will happen when pre-AGI
systems actually produce this much value; probably it will happen much earlier, as the potential
to automate large swathes of cognitive labour becomes apparent from impressive demos.

Once this “wake up” occurs, I expect investments in AI to scale up as quickly as possible until
they are worth $ trillions per year.

\(^2\) See footnote 48 of this CSET report; also informed by an unpublished memo by a CSET researcher.
\(^3\) NVIDIA R&D in 2021 was ~$4 billion, and they’re estimated to be 80% of the market share for AI chips.
I’m not sure how much of the $4 billion was spent on R&D for AI chips vs other R&D.
\(^4\) DeepMind annual spending is ~$1b. I’d guess total AI spend is 10 - 100X this.
\(^5\) Governments of powerful nations, leaders of large tech companies, militaries.
This section analyses the consequences of this fast-rising investment for the growth of 2020-FLOP used in the largest training run, g(2020-FLOP). I consider each of its three components separately: $ on FLOP, FLOP/$, and 2020-FLOP per FLOP. First, though, two sections about how I’m thinking about the dynamics of investment ramp up once the “wake up” has occurred.

In what follows, I’ll assume that we have had “wake up” by the time we start crossing the effective FLOP gap. This need not be true. But for my startpoint of “automate 20% of cognitive tasks”, I think it’s very likely. Automating 20% of cognitive tasks, on a naive calculation, would increase GDP by 25% which is ~$20 trillions.76

$ on FLOP for the largest training run

I break this component down into two subcomponents:

$ on FLOP for the largest training run = $ on FLOP globally * fraction of global FLOP on the largest training run

Let’s discuss each in turn.

$ on FLOP globally

I believe that, after “wake up”, the total amount of global FLOP will be bottlenecked by how quickly we’re able to design better chips and manufacture them rather than by willingness to pay.77

For this reason, I’m thinking primarily in terms of “how quickly can the world overcome production bottlenecks to produce more FLOP” rather than “how quickly will people increase their $ spending on FLOP”. In line with this, and as a part of a toy simplification discussed in this appendix, I’m interpreting “$ on FLOP globally” as meaning something like “number of chips globally”. Analogously, I’m interpreting FLOP/$ as “FLOP per chip”. This means, among other things, that I’m ignoring the fact that after “wake up” rising demand will increase the price of chips of a fixed quality.78

76 If the only effect [of automating 20% of cognitive tasks ] is to concentrate human workers on the remaining 80% of tasks, you’ll have 25% more workers per task on that remaining 80%. (100/80 = 1.25.) So the effective labour supply is 25% bigger; as a result you’ll accumulate 25% more capital and so GDP will increase 25%. (Here I make the standard assumption there are constant returns to labour and capital in combination.) This bottom line is too high in ignoring the non-cognitive labour that is not automated; this might reduce it by 2X. However, it is too low in ignoring the benefits of improving performance and throughput on the automated cognitive tasks, which could increase it by 2X.

77 I say more about this in an appendix.

78 See more.
How fast will the world be able to ramp up production of chips, when it’s doing so (roughly speaking) as quickly as possible? This is a thorny empirical question, which deserves much more attention than I’ve given it. This table summarises some weakly relevant quantities:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Growth rate</th>
<th>Comment on relevance to ( \text{g}(\text{on on FLOP globally}) ) after “wake up”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semiconductor revenue growth.</td>
<td>6 - 18%</td>
<td>Demand will be much higher after “wake up”.</td>
</tr>
<tr>
<td>TSMC revenue growth.</td>
<td>14 - 29%</td>
<td>Demand will be much higher after “wake up”. TSMC can potentially steal experts from rival companies.</td>
</tr>
<tr>
<td>Growth of munitions production at war time.</td>
<td>20 - 45%</td>
<td>Semiconductor supply chain is much more complex than munitions.</td>
</tr>
<tr>
<td>Time to build a fab. (Assumes that this equals the doubling time in the number of fabs.)</td>
<td>14 - 35%</td>
<td>My assumption about doubling times seems aggressive.</td>
</tr>
</tbody>
</table>

In a little more detail:

- **Semiconductor revenue growth.**
  - This grew at a rate of 6% from 2012-20 and a rate of 18% from 2018-20.
  - The higher rate might be a better indicator of the maximum capacity for growth, and thus of the growth after “wake up”. On the other hand, it could reflect a temporary effect or a spike in demand (that wasn’t reflected in an increase in supply).
  - My takeaway is a very weak pull to numbers in the ballpark 5%-20%.

- **TSMC revenue growth.**
  - Semiconductor revenue growth is constrained by demand. This is less true for an individual corporation like TSMC that can take market share from its rivals by growing its own output. So TSMC’s growth may mirror growth after “wake up” in that neither is constrained by demand.

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79. To simplify the main text, I elide the distinction between the *annual production* of FLOP, and the *total stock* of FLOP. Mathematically, it turns out that if the former grows at a constant rate, so does the latter. So here I estimate the growth rate of the former and use this as a proxy for the latter. However, this can lead us astray: if the growth rate of the former increases, there is a lag before the growth rate of the latter increases in step. So this distinction *is* included in the Full Takeoff Model (FTM), so is incorporated in the results of the sensitivity analysis in section 8. The effect is to make takeoff slightly slower.

80. Reminder: the growth rates here and throughout this report are *instantaneous growth rates*, not per-year growth figures. For example, if the instantaneous growth rate is 40% then the increase in one year is \( e^{0.4} = 1.49X \), which corresponds to a per-year growth of 49%.
This grew at 14% from 2011-21 and 29% from 2019-21.

- Its capital expenditures will grow at 35% from 2018-2022. This suggests the fast revenue growth is not mainly “more demand leads to higher prices for the same amount of real output”.

- On the other hand its number of employees only grew at 8% from 2009-2020 and at the same rate from 2018-2020. I’d be surprised if you could sustain 30% growth in output for very long without growing your employees faster than this.

- TSMC is potentially able to hire talent from other companies to help it scale, which may make these numbers too high. On the other hand, after “wake up” there will be (much) more willingness to pay and so I expect expansion efforts to be more aggressive even than TSMC’s recent expansion.

- My takeaway is a weak pull towards numbers in the ballpark 15-30%. I find this more informative than the semiconductor numbers.

- **Growth of munitions production at war time.**
  - This is an example of “growth of a specific industry’s output when there is very large demand”.
  - I looked at munitions output for countries involved in WW2. Growth rates were mostly between 20% and 45%, with the US as high as 80% (though from a much lower base as a fraction of their GDP).
  - I’d expect it to be harder to grow semiconductor output because its supply chain is notoriously complicated, probably much more so than munitions. E.g. this growth involved a lot of refitting existing factories to make munitions, which won’t be possible for cutting edge computer chips.
  - My takeaway is a weak pull towards numbers in the 20 - 30% range.

- **Time to build a fab.**
  - From a quick google, estimates vary from 2 - 5 years.\(^{81}\)
  - If this is also the doubling time for the number of fabs after “wake up”,\(^{82}\) that implies a growth rate of 14% - 35%. The assumption that the doubling time for fabs is the time it takes to build them feels aggressive to me.
  - My takeaway is a weak pull towards numbers in the 10 - 25% range.

- **Growth of the AI chip market.** It’s growing at a rate of ~30%. Naively, you’d expect growth to slow after it becomes a majority of semiconductors, but maybe the demand after “wake up” will allow it to continue at its current pace. Not convincing to me as its current growth is probably enabled by displacing production of other chips rather than creating additional production capacity.

I find the first quantity least informative, and the last three quantities similarly (un)informative as each other. My best-guess central estimate here is a growth rate (for $ on FLOP globally) of ~22%.\(^{83}\) I’d be surprised if the true figure was <10% or >40%. <10%, as well as falling outside

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\(^{81}\) This website says >2 years; this one says 3-5 years; this discusses an example that took 2.5 years.

\(^{82}\) The toy model here is that we’re able to leverage the expertise of each existing fab to build another one, and meanwhile train people up to work there.

\(^{83}\) Here’s how I arrived at this number. For the last three quantities my takeaway was a weak pull towards the ranges 15-30%, 20-30%, and 10-25%. Taking the average of the midpoint of each range gives \((22.5 + 25 + 17.5)/3 = 22\).
the range of the last three quantities, just feels very low for a world that’s trying its best to expand chip production. >40% implies a production doubling time of <21 months, which seems very quick given the staggering complexity of the production process and the requirement for workers with specialized skills.

When will this growth in $ on FLOP globally top out? The Full Takeoff Speeds Model currently assumes it continues until we’re spending 10% GWP annually on FLOP, ~$10 trillion today. Why 30%? Currently ~60% of GDP is paid to human wages; when AI plays a similar role to human brains we might pay a comparable % of GDP to rent chips to run AIs on. (I reduce the 60% to 10% to account for other costs of running AI systems and the possibility that we pay less for running AIs than we currently pay to human wages, e.g. due to being bottlenecked by non-cognitive inputs to production.)

Ok, I’ve explained how I’m thinking about the first component of $ on FLOP for the largest training run; now let’s discuss the second component.

Fraction of global FLOP on the largest training run

In my mind there are two important sub-questions here. First, how much room will there be in total to scale up the fraction of global FLOP used on the largest training run? This determines how many OOMs of the effective FLOP gap we can cross without even increasing the number of chips in the world.

Second, how quickly will we be able to scale up in this way? This determines how quickly we'll cover those OOMs.

If there’s many OOMs here, and we can cover them quickly, this could drive a very fast takeoff.

How much room to scale up the fraction of global FLOP used on a training run?

Drew Lohn from CSET has contracted with us and spent ~10 hours estimating the amount of FLOP currently available globally. Measured in terms of the FLOP we could perform by running the chips non-stop for a year for perfect utilisation, he estimates ~2e28 FLOP from discrete GPUs, ~4e28 FLOP from discrete+integrated GPUs, and ~2e29 if you include the large numbers of lower performance CPUs used in (e.g.) phones. In my view, these numbers are very uncertain and could easily be wrong by an OOM.

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84 In practice, in the Full Takeoff Model we typically get AGI before spending 10% GWP on FLOP.
85 As opposed to integrated GPUs, which are incorporated as part of a device like a laptop.
86 Drew Lohn actually estimated FLOP/s from the chips produced annually. I obtained these numbers by assuming the global stock of chips is 2X annual production.
I think a realistic maximum for the fraction of global FLOP that could be used in the largest training run is probably ~10%. That could involve, for example, using 30% of the world’s compute in a training run that lasts for 4 months. This already feels somewhat aggressive, but there will be economic incentives to combine compute together in big training runs and online learning will be a significant factor. Using 10%, the realistic maximum for FLOP used in the largest training run is currently 2e27 - 2e28 FLOP.

I believe the largest publicly available training run as of July 2022 is ~2e24, implying that the room for scale up is currently ~3 - 4 OOMs.

Of course, this quantity will change over time. I think the room for scale up will reduce over the next decade. Here’s an extremely rough estimate. Extrapolations of growth of the AI chip market imply growth of 15X by 2030; but the total quantity of global FLOP will grow by less, let’s say ~5X. Meanwhile, I expect the $ spent on the largest training run to increase by ~400X in that time. This implies that the room to scale will fall by ~2 OOMs to ~1 - 2 OOMs by 2030. For now, the Full Takeoff Speeds Model is assuming that this won’t change after 2030 until “wake up”.

For our purposes – estimating the time it will take to cross the effective FLOP gap – what matters is how much room for scale up will remain once we reach the startpoint. If we reach the startpoint around or after 2030, I expect ~1 - 2 OOMs room for scale. If we reach the startpoint before 2030, I expect somewhere between that and the current room for scale up (~3 - 4 OOMs); I’ll call it ~1 - 3 OOMs.

When will we reach the startpoint? The startpoint is measured in 2020-FLOP used for a training run. A $1b training run in 2030 would use ~2e29 FLOP, based on the bio-anchors extrapolations of FLOP/$ and 2020-FLOP per FLOP. So people with startpoints lower than 2e29 2020-FLOP should expect us to reach the startpoint before 2030.

<table>
<thead>
<tr>
<th>Startpoint, 2020-FLOP</th>
<th>Reach before 2030?</th>
<th>Room left for scale up when we reach the startpoint</th>
</tr>
</thead>
</table>

---

87 This, together with my earlier assumption that we might ultimately spend 30% GWP on FLOP, implies that we’d spend 3% GWP on FLOP for the largest training run. This is a lot; Bio Anchors caps this at 0.25%. But remember that an AGI training run will be happening in a world where AI systems are already generating $10s trillion per year.

88 I believe these were the requirements for PaLM.

89 The 5X is for $ on “discrete GPUs”, which can still grow significantly as a fraction of fab production. If we used the broader category of all GPUs and CPUs, I’d expect lower growth.

90 It’s estimated that PaLM cost ~$10m to train. My median guess is that the largest 2030 spend is ~$4b, in line with Bio Anchors’ projections. That’s a 400X increase. (I believe AI companies can buy FLOP ~3x cheaper than if I were to rent GPUs on the cloud, so in practice it may cost them more like ~$1b.)

91 In the FTM the median room for scale in 2030 is ~1 OOM because it uses the “discrete GPU” category as its median.

92 I.e. it assumes that after 2030 training runs will scale up at the same pace as global FLOP until “wake up”.

93 See calc.
How quickly will we increase the fraction of FLOP used on a training run?

A naive approach here is to anchor off the scale up over the last decade. I roughly estimate that, in 2012-2018, the fraction of AI chips used in a training run grew at a rate of ~150%. That’s an OOM every 18 months. If we scale similarly fast after “wake up”, we’ll reach the maximum in just a few years.

However, I believe that the engineering barriers to further scaling will be much more significant than in the past, e.g. in building the infrastructure for training to be efficiently distributed over many chips in parallel. I don’t know how long these problems will take to overcome; this is an area where further empirical research is needed.

Another big uncertainty is how hard it will be to adjust manufacturing processes to produce AI specialised chips rather than other chips. I’d guess that this will be easier for discrete GPUs than for CPUs.

My current wild guess is that after “wake up” enough effort will go towards this that we will scale up at a pace of an OOM every ~2 years. That amounts to a growth rate in the fraction of chips on the largest training run of ~110%. But it’s conceivable to me that we will do 2 OOMs in one year with sufficient effort (~450%), and conceivable that we can only do an OOM every 4 years (~55%).

The strategic importance of scale up

This dynamic of scale up could drive an extremely fast takeoff. If the effective FLOP gap is small (<=2 OOMs), room for scale up is large (>=2 OOMs) and scale up happens quickly, we may cross the effective FLOP gap extremely quickly. Concretely, this might look like one big actor making a deal with TSMC or NVIDIA to buy the majority of their output for a year, using this to 100X the largest training run so far, and thereby blasting through the OOMs for FLOP around which AI capabilities are most concentrated.

There’s a counter-intuitive implication here. One sure-fire way to avoid this scenario is to scale up training runs before we reach the startpoint (before AIs become really capable). That way there’s no room left to scale when we hit the startpoint. Of course, there are considerations

<table>
<thead>
<tr>
<th>&lt;2e29 2020-FLOP</th>
<th>Yes</th>
<th>1 - 3 OOMs</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;2e29 2020-FLOP</td>
<td>No</td>
<td>1 - 2 OOMs</td>
</tr>
</tbody>
</table>

94 In 2012 - 2018 training FLOP increased by 300,000X, corresponding to a growth rate of 210%. \[\exp(2.1\times 6) \approx 300,000\]. To get the growth in the fraction of FLOP we need to subtract out the growth of total FLOP. Let’s subtract 30% for growth of FLOP/$ and 30% from growth in $ on FLOP, leaving 150%. [Do this again with Jaime’s numbers for the longer period?]

95 \[e^{(150\times 1.5)} = 9.5\].

96 \[e^{(1.1\times 2)} = 9\].
which argue in the opposite direction, so this doesn’t imply that we should in fact scale up training runs today.

**Summing up $ on FLOP for a training run**

There are two sources of growth for this quantity: increasing the $ on FLOP globally and increasing the fraction of FLOP on a training run. Mathematically:

\[
g(\text{FLOP for a training run}) = g(\text{FLOP globally}) + g(\text{fraction of FLOP on a training run})
\]

My very rough best guesses are:

- \( g(\text{FLOP globally}) = \sim 22\% \), probably between \sim 10\% and \sim 40\%.
- \( g(\text{fraction of FLOP on a training run}) = \sim 75\% \), with room for 1 - 4 OOMs of growth here in total (more likely towards the lower end).
- So \( g(\text{FLOP for a training run}) \) is initially \sim 97\% and later \sim 22\%.

This concludes the discussion of how rising AI investment after “wake up” might affect $ on FLOP; the next section analyses the same question for FLOP/$.

**FLOP/$**

How might rising AI investments after “wake up” affect FLOP/$? The most salient mechanism is that they increase R&D efforts towards making better chips and increasing FLOP/$. In 2000 - 2020, real $ inputs to hardware R&D have grown slowly at \sim 4\% a year. Faster input growth after “wake up” should accelerate growth of FLOP/$.

How can we estimate the size of this effect? My approach is to:

1. Use historical data to fit an economic model relating hardware R&D inputs to increases in FLOP/$. The fitted model implies that each doubling of cumulative inputs leads to 5.2 doublings in FLOP/$.
2. Forecast how quickly inputs will grow after “wake up”.
   a. I guess that annual R&D inputs might grow at a rate of 17\%.
   b. If cumulative inputs grew at 17\%, the model implies that FLOP/$ will grow at 5.2 * 17\% = 88\%, ~9 month doubling.
   c. But, importantly, I’m currently forecasting that cumulative inputs will initially grow at their current pace of \sim 4\%, and that their growth rate will gradually increase to

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97 For example, scaling up training runs today might cause “wake up” and thus bring AI timelines forward by many years, reducing time for safety work, movement building, and other preparations.
98 As mentioned above, I’m excluding the effect of “bidding up the price of compute”.
99 I compare my $ on FLOP forecasts with those from Bio Anchors in this appendix.
100 Remember I’m ignoring the possibility that compute prices are significantly bid up, and using a simplification in which all increases in FLOP/$ are due to hardware R&D (and all increases in $ on FLOP and due to scaling up production of chips).
17%. This implies growth in FLOP/$ will gradually increase from its recent rate of doubling every ~3 years.

I’ll discuss each step in turn.

Economic model relating hardware R&D inputs to outputs

I’m using what economists typically consider to be the best model of R&D - the semi-endogenous growth model (SEG). Its distinctive advantage, compared to the alternatives, is that it quantifies the extent to which ideas are getting harder to find, or the extent to which there are diminishing returns to doing more research.

One way to understand the core of the model is as saying:

Each time cumulative inputs double, the output metric doubles \( r \) times

So for each \( x\% \) increase in cumulative R&D inputs, the output metric will increase by \( r^*x\% \). The inputs could be measured in $ or in (quality-adjusted) researcher-years; the output metric can similarly vary.

Estimating \( r \) for hardware

What happens when we fit a SEG model to this data on hardware inputs and outputs? I did this, taking the inputs to be real $ invested in semiconductor R&D and the output to be measured FLOP/$.

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101 To my mind most of the alternatives are pretty implausible, so being “the best” is means something like “being broadly consistent with the empirical evidence and not having significant noticeable flaws” rather than being well confirmed.

102 As such, it’s the model used in the famous paper by that name.

103 How does this relate to the standard presentation of the theory in terms of the stepping on toes parameter lambda and the fishing out parameter phi? \( r = 1 / (1 - \text{phi}) \). If lambda < 1 then we should adjust our description of what the core of the model to ‘For each \( x\% \) increase in cumulative effective R&D inputs, the output metric will increase by \( r^*x\% \), and add that the effective input in each year is input^lambda.

104 Then in what sense are “ideas are getting harder to find”? Somewhat confusingly, economists define “one idea” as a 1% increase in the output metric. Early R&D models implied that every such increase would require the same absolute increase in cumulative inputs. But this model implies that the cumulative input increase required grows over time. The smaller \( r \), the more effort must increase from one idea to the next. More precisely, compare the effort needed to double the output metric on two consecutive occasions (e.g. increasing it from 5 to 10, and from 10 to 20). The effort for the second doubling is greater by a factor of \( 2^{1/r} \).

105 I got my dollar input data from Are ideas getting harder to find. I took their numbers for nominal $ hardware investment, using their category 'PatentNarrow (include equipment)'. I extend their data after 2015 based on recent growth in R&D inputs (rows 41 - 46). Then I adjust these nominal $ inputs for inflation. See calcs here.

106 I got my FLOP/$ from the Bio Anchors appendix, top of p.29. I read off FLOP/$ values for 1970, 2008 and 2018, see rightmost columns here. (Note, this data doubles-up as data on the growth of FLOP/s per
The following table summarises the calculation:

| Time period       | g(cumulative inputs) | g(FLOP/$) | Estimated hardware returns, $r$
|-------------------|----------------------|-----------|------------------------
| 1970 - 2018       | 7.9%                 | 53%       | 6.7                    |
| 1970 - 2008       | 8.5%                 | 61%       | 7.1                    |
| 2008 - 2018       | 5.4%                 | 23%       | 4.3                    |
| 2006 - 2022 (GPUs)| 5.4%                 | 28%       | 5.2                    |

*If you prefer thinking in terms of doubling times rather than growth rates, use this converter.*

The calculation is simple. For each time period, $r = g(\text{FLOP/$}) / g(\text{cumulative inputs}).$

So the data suggest that returns to hardware R&D were very good from 1970 - 2008, with each doubling of cumulative inputs leading to 7 doublings for FLOP/$. Returns have been less good recently, with each doubling of inputs driving a little over 4 doublings of FLOP/$. Though if you focus on GPUs, returns look a little better.

I’ll take the GPU figure, 5.2 as my median estimate of the current value of $r$. It could be higher if there’s a reversion to the longer-run historical returns, or it could be lower if returns are worse today than in the period 2008-2018. Additionally, the Full Takeoff Speeds Model assumes that $r$ decreases towards 0 as FLOP/$ approaches physical limits.

(There is a subtlety in accounting for the *stepping on toes* effect, where doubling the real $ investment in a given year less than doubles the progress that year due to barriers to parallelising research. I discuss this in an appendix.)

How quickly might hardware R&D inputs grow after “wake up”?

How quickly could we grow real $ inputs to R&D when we’re trying very hard after “wake up”?\(^{107}\)

This is another input where the best I have been able to do is point to weakly relevant empirical trends. Here are some:\(^{108}\)

- **Growth of US federal R&D around WW2.**
  - The US made a notable effort to expand R&D during and after WW2, so may be an indicator of how quickly R&D efforts can be expanded.

\(^{107}\) I’m explicitly not including the $ value of AI systems used to do hardware R&D in this section. That is, I’m forecasting the growth in the real inputs of human labour and non-AI capital. The next section gives a separate treatment of the effect of incremental AI automation on the pace of R&D progress.

\(^{108}\) All figures give the growth of real $ inputs unless stated otherwise in a footnote.
Examples:
- Defense R&D grew at 15% from 1950 to 1962.
- Civilian R&D (which includes many sectors) grew at 17% from 1950 to 1966, though probably total R&D grew more slowly than this.
- Space R&D grew at 36% from 1954 to 1966, though from a small base.
- Total federal R&D grew at 11% from 1937 - 1953 (I think this number is sketchy).

I think the financial incentive for hardware R&D will be greater after “wake up”, but it will be growing from a higher base. My takeaway is a weak anchor to 10-30%.

Historical growth of semiconductor R&D inputs.
- These grew at 10% initially, and this growth rate has gradually declined over time (see above).
- Again, the incentive to grow R&D will be much greater after “wake up”, but it will be growing from a much larger base. I’m not sure how these net out.

Historical growth rate for other areas of R&D.
- Are Ideas Getting Harder to Find contains estimates of the growth of inputs for a number of fields; numbers range from 2% to 10%.\textsuperscript{109}
- I’d expect hardware inputs to grow at or above the higher end of this after “wake up”.

Recent growth of R&D in hardware companies whose revenues are growing quickly.
- These provide evidence about how quickly hardware companies grow R&D when there is lots of demand for their output.
- Examples:
  - ASML R&D grew at 17% from 2016 - 2021.
  - NVIDIA R&D grew at 19% from 2016 - 2021, and 14% from 2005 - 2021.
  - TSMC R&D is growing at 4 - 5%. (Their capital costs are growing much more quickly than this.)
- My takeaway is a weak anchor to ~20%, as the demand will be higher still after “wake up”.

Growth of R&D relating to covid-19.
- From January 2020 to June 2020, the number papers published related to covid grew by almost 2 OOMs.\textsuperscript{110}
- While I doubt the new authors were adding nearly as much value as existing experts, and the initial base is clearly much smaller for covid, this was an update for me towards scientists’ ability and willingness to pivot to new fields.

[Any other ways ppl think of to inform a guess at this?]

\textsuperscript{109} The growth rate of real $ are probably slightly higher, as the paper uses a different measure of inputs. Its units are “salary of a high-skilled worker”, and I think a “high-skilled worker” is operationalised as a graduate. So if graduate salaries have gone up in real terms, the paper’s growth rates will underestimate growth as measured in real $.
\textsuperscript{110} See the “Sell it” section of this Matt Clancy substack post.
My best-guess central estimate here is an input doubling time of ~3.5 years, which is a **growth rate of ~17%**. This is a little higher than the trends above, but I expect there to be huge demand once $T$ trillion training runs are on the table, and sufficient numbers of high-skill people who can add value (e.g. who currently work in finance, physics, other areas of materials science) for the field to double every 3 - 4 years. I’d be surprised if this <8% as 8% is not unusual for the growth of R&D fields. Similarly I’d be surprised by >35% because i) even space R&D did not grow faster than this from a small base in the 1950s, and ii) doubling the semiconductor R&D field in just 2 years (35% growth) feels like a pretty tall order.

When will this growth top out? The Full Takeoff Speeds Model currently assumes it continues until we’re investing 3% GWP annually, ~$2.5 trillion today. That’s ~35X larger than today’s figure of $70b, allowing for ~20 years of 17% growth.

Why a ceiling of $2.5 trillion? One anchor is that annual semiconductor revenues today are ~$550b while R&D is ~$70b: a ratio of six. If annual semiconductor revenues are growing towards being worth $30tr, as I assume above, then that same ratio implies that R&D should be growing towards $5tr. I’m reducing that to $2.5trillion because total global R&D spend today is only ~$2 trillion.

**Growth of FLOP/$ after “wake up”**

If cumulative inputs grow at 17%, and $r = 5.2$, FLOP/$ will grow at $5.2 \times 17\% = 88\%$. That’s roughly a 9-month doubling.

There is an important caveat. There is a difference between annual and cumulative inputs. If annual inputs suddenly start growing at 17% (rather than their current ~5%), there is a lag before cumulative inputs grow at the new faster rate. So if faster growth of annual R&D inputs coincides with the startpoint, there will be a >10 year lag before FLOP/$ grows at the new quicker rate.

This dynamic – the distinction between annual and cumulative inputs – is modelled explicitly in the Full Takeoff Model (FTM).

I worry the FTM is conservative for modelling the response to “wake up” merely via a faster growth rate in annual hardware R&D spending, rather than also including a one-time jump in spending. A one-time jump would reduce the lag before FLOP/$ grows at the new quicker rate, and could significantly reduce the time crossing the effective FLOP gap.

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111 If you’re going to spend $1 trillion on FLOP for a training run, it’s worth spending $500 billion on R&D to double FLOP/$. Current semiconductor R&D is only $70 billion.

112 In fact, the growth rate of cumulative inputs gradually increases from the old growth rate of annual inputs to the new growth rate of annual inputs. Also, it turns out a larger “stepping on toes” effect causes a longer lag before output grows at the new pace, see more [here](#).
This concludes the discussion of how AI investments after “wake up” might affect trends in FLOP/$. Now I turn to how they might affect 2020-FLOP per FLOP.

How big is the relevant bucket of hardware R&D?

The Full Takeoff Model (FTM) uses ~$80b as the hardware R&D spend in 2022, the figure for semiconductor R&D spend.

But you might choose to use a larger figure if you include the potentially-larger buckets of computing and electronics or materials science. The logic for inclusion would be that, in the long run, semiconductor R&D progress is reliant in progress in these broader areas. Using a This would leave less room for investment to hardware R&D to grow before reaching a cap; it might also change the historical growth rates of R&D spending.

Alternatively, you might choose to use a smaller figure if you restrict to hardware R&D specifically targeted at improving chip design for AI use-cases. This would leave more room for R&D spending to grow, and probably imply recent historical growth of R&D spending was higher.

In reality, the smaller bucket is probably more relevant over shorter timescales (where existing node sizes can be specialised for AI algorithms) but the larger bucket will become increasingly relevant over longer time periods (where entirely new computing paradigms must be invented). This means the FTM might underestimate R&D progress in the short term but overestimate progress in the long term.

This model implies that hardware progress will continue to slow down before “wake up”

The above data show that growth of FLOP/$ has slowed over time. The semi endogenous growth model I’m using predicts some but not all of that slowdown, because the growth of cumulative hardware inputs has also slowed over time (but not by as much). This gives us some reason to prefer it to a simple trend extrapolation.

But, in the near term, it seems like the growth of cumulative R&D hardware inputs will continue to slow. So the semi endogenous growth model predicts that the growth of FLOP/$ will continue to slow during the 2020s. In fact, it predicts its current growth rate is lower than its

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113 This chart puts Computing and Electronics at ~20% of global R&D spend, which would be 0.2*$2tr = $400b.

114 Why? Historically, annual R&D inputs grew quickly at a rate of ~10%. Recently, they’ve grown slower, at a rate of ~4%. As a result, cumulative R&D inputs have gradually been growing more slowly over time, with their growth rate moving gradually down from 10% towards 4%, currently at ~6%. If annual R&D inputs continue to grow at ~5% (as I’m forecasting before “wake up”), then the growth of cumulative R&D inputs will continue to slow from 5% to 4%.
recent average (i.e. the averages reported in the table above) because the current growth of cumulative inputs is lower than the recent average growth rate.

If I had used a smaller bucket of R&D, restricted to designing AI-specialised chips, this model might have the opposite conclusion. Plausibly, the cumulative hardware inputs in this narrower bucket will grow more quickly in the near future than the recent past. This is another way in which the FLOP/$ forecast of this model could be considered to be conservative.

But aren’t we approaching the physical limits of the current paradigm?

It was beyond the scope of this report to do an investigation into the details of how long progress could continue within the current hardware paradigm, and how promising new paradigms are. Instead this report takes a zoomed out “outside view” approach to forecasting hardware progress, extrapolating the observed relationship between inputs and outputs.

Would investigating the details of the current paradigm imply that this report overestimates future hardware progress? Plausibly, but it’s not obvious to me.

- As mentioned above, the report assumes that hardware innovations are getting harder to find, with more research effort required for each successive doubling of FLOP/$. This captures the intuition that progress will become more difficult as we approach the end of the current paradigm.
- In addition, the FTM assumes that the rate at which hardware innovations become harder to find itself increases. The returns diminish increasingly steeply as we make more hardware progress. Mathematically, this corresponds to reducing $r$ as FLOP/$ increases. This is an additional conservative adjustment to naive trend extrapolation.
- It’s possible that the rate of progress will be faster in a new paradigm, rather than slower.

2020-FLOP per FLOP

The approach here is the same as in the last section. This time I use inputs to software R&D rather than hardware, and measure output as 2020-FLOP per FLOP rather than FLOP/$.

In particular I:

1. Use historical data to calibrate an economic model relating cumulative inputs to output.
   a. This time the data is significantly more uncertain, especially the output data.
   b. Ultimately I assume that each doubling of software inputs drives ~1.25 doublings of 2020-FLOP per FLOP.
2. Forecast how quickly inputs will grow after “wake up”. I guess that these will grow at ~25%, slightly faster than their recent rate of 20%. This implies that 2020-FLOP per FLOP will also grow at ~31%.\(^\text{115}\)

\(^{115}\) \(1.25 \times 25\% = 31\%\).
Economic model relating software inputs to outputs
As last time, the core of the model can be expressed as:

*For each x% increase in cumulative R&D inputs, the output metric will increase by r*x%.*

Our inputs are real $ invested in software R&D; our output metric is 2020-FLOP per FLOP.

The data for software inputs and outputs is not comparably good as for hardware. On the input side, the best I know of are the numbers from Tamay Besiroglu’s dissertation. He uses data on the number of authors of papers in three subfields of ML as a proxy for the number of researchers. After some adjustments, we end up with following estimates: ¹¹⁶

<table>
<thead>
<tr>
<th>ML subfield</th>
<th>Growth in the annual inputs to software R&amp;D, 2012 - 2020</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer vision</td>
<td>20%</td>
</tr>
<tr>
<td>Natural language processing</td>
<td>36%</td>
</tr>
<tr>
<td>Graphs</td>
<td>42%</td>
</tr>
</tbody>
</table>

My guess is that the real growth of inputs in this period is lower, mostly because these growth rates seem very high and I think this estimate is very uncertain. ¹¹⁷ (If these numbers are correct, it suggests I’ve been too conservative with my estimates earlier in this section.)

On the output side, the ideal situation would be to have trustworthy estimates of how many FLOP would be needed to train AGI at different points in time. This would be a direct estimate of the growth in 2020-FLOP per FLOP.

In actual fact, the best I’m aware of is to track the training compute needed to achieve a fixed score on a specific benchmark over time. The best example of this type of analysis that I know for AI is AI and Efficiency, which finds that runtime compute needed for a fixed performance on AlexNet halved every 16 months (growth rate of 52%) between 2012 and 2020. ¹¹⁸

¹¹⁶ Tamay multiplies (an estimate of) the number of paper authors from each country by the average salary of scientists in that country, measured in nominal $. I adjust these numbers downwards somewhat to account for inflation. So the R&D inputs in the table are measured in real $. See my calcs here.

¹¹⁷ Uncertain for at least two reasons. Firstly, I don’t expect the “number of distinct authors” to correlate perfectly with “number of full time researchers”. Secondly, the attempt to adjust for the quality of the researchers seems unconvincing: they multiply by the salary of the researcher’s country but I’d expect that within each country the new researchers are mostly young.

In addition to the numbers looking high and being uncertain, there’s another reason I think the true growth is probably lower. The growth rate of cumulative inputs will be lower than the growth rate of annual inputs, assuming that growth of annual inputs was slower before 2012. And it’s growth of cumulative inputs that matters for this economic model linking R&D inputs to outputs.

¹¹⁸ I discuss non-AI software trends here.
I believe that other benchmarks show similar or faster rates of software progress (with doubling times of 1 - 2 years) when this has been measured.\footnote{For example, table 2 of OpenAI’s paper shows similar or faster software gains on other select tasks as on ImageNet, and people I’ve spoken to suggest recent progress on language models has been faster than the ImageNet progress.}

Estimating $r$ for software

If we naively combine the $g(2020$-FLOP per FLOP) = 52% estimate from AI and Efficiency with Tamay’s estimate of the growth rate of real $\$ $ inputs to computer vision, we get $r = 52\%/20\% = \sim 2.5$.

If we use this as our central estimate we are assuming that the software progress on ImageNet will match that on AGI (in expectation). But in Bio Anchors, Ajeya writes:

“... researchers have strong feedback loops on ImageNet, and I would expect them to be less efficient at reducing computation costs for something which has never been done before, such as “training a transformative model.”

Another reason for the same adjustment is that we might imagine AGI software progress is the average of all areas of AI, and that the areas where we’re measuring progress have faster progress than the areas we’re not interested in measuring. On the other hand, some algorithmic progress seems to reduce the compute needed large training runs more than the compute to smaller training run, suggesting the compute needed to train AGI may be falling more quickly than that for ImageNet.

Ultimately, I follow Bio Anchors and assume ~halve the rate of software progress as observed in ImageNet.\footnote{She replaces the observed 16 doubling time with a 2 - 3 year (i.e. \sim 30 month) doubling time.} For now I will remain consistent with Bio Anchors and make an equivalent adjustment. This halves my estimate to $r = 1.25$.

Using $r = 2.5$ would bring forward AGI timelines by \sim 3 years as well as making takeoff faster. In the Monte Carlo I use large uncertainty bounds for this parameter: 0.8 - 5.

How quickly might software R&D inputs grow after “wake up”?

I don’t have much to add here to the analogous section for hardware. In that case, my central estimate was 17%. I want to use a higher number here, for two reasons:

1. The AI software sector is growing from a smaller base. I guessed AI software spend is \sim $10-20 billion, vs $70 billion for semiconductor R&D.
2. The AI software sector is apparently already growing faster than 17%: the numbers above range from 20% to 42%. I expect that after “wake up” inputs to software will grow as fast or faster as it is currently, based on the huge demand.
a. I’m going to anchor to the 20%, rather than the 42%, because I used the 20% to estimate $r$.\textsuperscript{121}

Based on the above, my central estimate here is that real \textbf{\$ invested in software R&D will grow at a rate of $\sim$25\%}. For similar reasons to above, I’d be surprised if this is $<15\%$ or $>40\%$.

This implies a central estimate of $g(\text{2020-FLOP per FLOP}) = 1.25 \times 25\% = 31\%$. This is very similar to the Bio Anchors extrapolation of $\sim 28\%$; just a touch higher as I expect inputs to grow somewhat faster than they are currently.

\textbf{Summing up}

\begin{align*}
g(2020-\text{FLOP}) &= g(\$ \text{ on FLOP in training run}) + g(\text{FLOP/$}) + g(\text{software}) \\
g(2020-\text{FLOP}) &= g(\text{fraction FLOP on training run}) + g(\$ \text{ on FLOP globally}) + g(\text{FLOP/$}) + g(\text{software}) \\
g(2020-\text{FLOP}) &= \text{Initially $\sim 75\%$, then $\sim 0\%$ + $\sim 22\%$ + $\sim 25\%$ (increasing over time towards $88\%$) + $\sim 31\%$} \\
&= \text{Initially $\sim 153\%$, then $\sim 78\%$ (increasing over time towards $141\%$)} \\
&\text{\quad $\sim 0.7$ OOMs/year, then $\sim 0.3$ OOMs/year (increasing over time towards $\sim 0.6$)} \quad \text{\textsuperscript{122}}
\end{align*}

This section analysed the effect of fast-rising AI investments on the speed crossing the effective FLOP gap, summarised in the diagram above. This has implications for takeoff speeds and for timelines.

The next section analyses the effect of incremental AI automation on speed crossing the effective FLOP gap. In doing so, it introduces the theoretical machinery for calculating new metrics for takeoff speed that relate to the \textbf{number of AIs} and their effects on GDP.

\textsuperscript{121} If I’d used the 42\% to estimate $r$, I’d probably be using a lower value for $r$. (Though it’s possible that software progress has simply been twice as fast in Graphs than in Computer Vision.)

\textsuperscript{122} \textbf{Link to diagram.}
5. AI automation

*I think this section of the long summary summarises the important takeaways from sections 5 and 6 in a just few pages. I’d only read this section if you really want to understand the math behind the automation models I’m using further, but aren’t familiar enough with growth economics already to read the mathematical description of the Full Takeoff Model.*

Summary

This section analyses the effect of incremental AI automation on takeoff speeds.

By “incremental” I just mean that I model AI automation as a *continuous* process of automating more and more tasks, without any discrete “jumps” in which AI suddenly automates lots of cognitive tasks in one fell swoop. I do not mean to imply that this process happens slowly; in fact it may happen very quickly. The speed depends on the size of the effective FLOP gap and how quickly we cross it.

As a recap, the two key inputs to takeoff speed are the **effective FLOP gap** (measured in 2020-FLOP) and our speed crossing the effective FLOP gap, $g(2020\text{-FLOP})$. Section 3 described this framework and laid out evidence informing the effective FLOP gap, and section 4 analysed the effect of rising AI investment on $g(2020\text{-FLOP})$.

This section analyses the effect of AI automation on $g(2020\text{-FLOP})$. As in section 4, I’ll analyse each of the three components of 2020-FLOP separately. This time I’ll take them in reverse order: **2020-FLOP per FLOP**, then **FLOP/$** , and finally **$ on FLOP**.

For each component, I will use a certain economic model to analyse the effect of AI automation.\footnote{\label{fn:task-based-model}I’ll use a task-based model, and explain it below.}\footnote{\label{fn:cobb-douglas}This is the Cobb Douglas version of the task-based model.} In this section I use a simple version of the model which excludes certain bottlenecks.\footnote{Bottlenecks are discussed in section 6.} Then I’ll explain how we can calculate some additional metrics of takeoff speeds using our model of AI automation. In particular, metrics relating to the number of AIs and their effects on GDP.

The key takeaways from all this are:

- **AI automation** causes growth of the three components to accelerate continuously. By the time we reach full automation of cognitive labour (AGI), they can double in months or faster.
  - This includes a feedback loop where more 2020-FLOP leads to training better AIs and running more AIs, which in turn allows us to produce more 2020-FLOP.
• Initially, when a small fraction of cognitive tasks have been automated, AI automation has a small effect on the growth of the three components compared to the rising investment discussed in section 4.
  ○ The effect on FLOP/$ and software becomes significant, relative to rising human investment, when ~25% of cognitive tasks have been automated.
  ○ The effect on $ on FLOP becomes significant when ~45% of cognitive tasks have been automated.
  ○ This doesn’t account for bottlenecks, which would increase these percentages somewhat.
• Unfortunately, I’m not aware of a simple, analytically tractable way to calculate overall takeoff speed metrics given the feedback loops involved here. My approach instead is to simulate the model and do a sensitivity analysis, which I’ll present in section 7.

This section is more technical than other sections. Many readers will prefer to skip to the next section.

2020-FLOP per FLOP

2020-FLOP per FLOP increases because of software research. I’m going to model incremental AI automation as a continuous transition from “world 1”, where humans do ~all the software R&D, to “world 2”, where AIs do ~all the software R&D. First I’ll discuss the dynamics of world 1; then those of world 2. Lastly I’ll explain how I’m modelling the transition between the two.

World 1

We’re currently in a world where software research is overwhelmingly done by human workers. Let’s call this world 1. In section 4, I forecasted how quickly these human inputs to software R&D might grow after “wake up”, and what effect this might have on 2020-FLOP per FLOP.\textsuperscript{125}

\textsuperscript{125} I guessed that each doubling of cumulative software R&D inputs would cause 2020-FLOP per FLOP to double 1.25 times. I measured in the inputs as real $ spent on R&D.
World 2

Once we have billions of AGIs – remembering that AGI is AI that can automate 100% of cognitive tasks – we’ll be in a world where software research is overwhelmingly done by AIs. Let’s call this world 2.

Ultimately, we’ll model the transition from world 1 to world 2 by assuming progress is driven by a combination of AI and humans.
Before this, let’s think about what will happen to the level of software world 2?

We can understand this dynamic by answering two questions:
1. How long does the first doubling of software (i.e. 2020-FLOP per FLOP) take in world 2?
2. How do the lengths of the software doublings change over time in world 2?

How long is the first software doubling?

The answer to the first question depends on i) how many human researcher-years are needed to double software when we first get AGI, and ii) how many AGIs you can run (where each AGI is as productive as a human per day).  

Here’s a very rough estimate of (i). If there are 20,000 high-quality human researchers on software today and software doubles every ~2 years then it currently takes 40,000 researcher years to double 2020-FLOP per FLOP. Let’s assume this is ~100X higher by the time we get AGI due to diminishing returns from the research that happens before then. That implies ~4 million researcher-years to double software when we get AGI.

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126 A more precise formulation of (ii) is: how many human workers you’d need to make the same software progress per day as the collection of AIs you can run. This formulation reflects the fact that there may be a variety of different AIs doing different software tasks, rather than just AGIs doing all of them. (Indeed, this is what happens in the Full Takeoff Model!)
127 DeepMind has 1000 employees, and I earlier assumed that total AI software input was 20X that of DeepMind. (The 20X is a guess, and someone could probably easily get a better number.) Facts that are potentially relevant to a more careful estimate: 200 new AI PhDs in 2020; 80,000 AI journal publications in 2020.
128 In the ImageNet example, 2020-FLOP per FLOP doubled every 16 months.
129 100X corresponds to cumulative research inputs growing by 100X by the time we develop AGI, which could be from growing 23% per year for 20 years before we develop AGI. e^(0.23*20) = 100. I’m using a model in which the effort needed to double software is proportional to the total cumulative input so far.
To estimate (ii), suppose you trained AGI with $1 \times 10^{32}$ 2020-FLOP, the training run took 4 months, afterwards you used 10% of your training compute to run AGIs doing software research, and running an AGI required $1 \times 10^{16}$ 2020-FLOP/s.\textsuperscript{130} With these conservative assumptions,\textsuperscript{131} you’ll have 100 million AGIs doing software research and so the first software doubling will take ~1 months.

Our estimate here could easily have been more aggressive:

- If instead AGI requires $1 \times 10^{36}$ 2020-FLOP to train but has the same runtime requirements (e.g. due to a long horizons), the first software doubling will be OOMs quicker as we’ll have more 2020-FLOP to run AGIs.
- If AGI has significant “one-off” productivity advantages over humans for R&D (run faster in serial time, no sleep or leisure, better motivation and coordination, all AGIs are copies of the most productive AGI) then this will speed up software progress. My current \textit{guessestimate} of these advantages for R&D is ~60X.

A more aggressive estimate by 10X would naively implies software doubling in a couple of weeks, though at that point bottlenecks from computational experiments become salient.

The point here is not to trust these exact numbers.\textsuperscript{132} It is to see the way in which the time for the first software doubling depends on the AGI’s training compute, AGI’s runtime compute, and the amount of software research that has happened before AGI. It is secondly to make plausible the idea that the initial software doubling in world 2 could happen in months or much less.

How do the lengths of the software doublings change over time?

In world 2, the annual inputs to software R&D are proportional to the 2020-FLOP used for this purpose.\textsuperscript{133} This means that there is a very direct feedback loop between the inputs and outputs of software research, unlike today. Doubling the software R&D output metric also doubles the input to software research.

\textsuperscript{130} I.e. $1 \times 10^{16}$ 2020-FLOP are needed to do as much quality-adjusted cognitive labour as a human does in 1 second.

\textsuperscript{131} Small training 2020-FLOP for AGI; a large runtime compute for AGI; only 10% of compute on software work; ignores the fact that inference is more efficient than training; ignores the possibility of AIs perform some tasks much more compute efficiently than human brains (e.g. like calculators are OOMs more compute efficient than human brains at arithmetic, or facial recognition systems are OOMs more efficient at recognising faces, or GPT-3 is OOMs more compute efficient at writing poems).

\textsuperscript{132} In fact, the first doubling time depends on many interacting factors (like "how much will returns to software research have diminished by the time we get to AGI?") and is hard to estimate analytically. Ultimately, I get around this by simulating the system and doing a sensitivity; software doubling times are almost always extremely fast (<6 months) by the time we have AGI, even if there isn’t a “software singularity” (discussed below). But the simulation omits bottlenecks from computational experiments.

\textsuperscript{133} We can in principle distinguish between 2020-FLOP for \textit{training} AGI (my main focus thus far in the report) and the 2020-FLOP for \textit{running} AGI. It is possible that they grow at different rates, e.g. if a new algorithm reduces training FLOP but not runtime FLOP. For our present purposes, it is the \textit{runtime} 2020-FLOP that concerns us. For ease of exposition, I won’t explicitly distinguish between these two unless it is relevant. [Describe what Full Takeoff Model does here. TODO]
The feedback loop is:

*Better software* → *more 2020-FLOP* → *more software R&D* → *better software*

It turns out that, with this feedback loop, there are two broad possibilities.

1. **Software singularity - quicker and quicker doublings.** If returns to software R&D exceed a certain threshold, the feedback loop is so powerful that there’s a “software only singularity”. The level of software, quantified here as 2020-FLOP per FLOP, grows faster and faster, theoretically going to infinity in finite time. And this happens even using a fixed quantity of physical FLOP to run the AIs. In practice, of course, the software returns become worse before we go to infinity and we move to the second possibility.

2. **Software fizzle - slower and slower doublings.** If returns to software R&D are below a certain threshold, the level of software grows more and more slowly over time, assuming a fixed quantity of physical FLOP. (If the amount of physical FLOP is in fact growing increasingly quickly, then the level of software can do the same. But software progress is reliant on the growth of physical FLOP.)

Which possibility will obtain? It turns out that there is a software singularity just if \( r > 1 \), where \( r \) is defined as in section 4:

*For each doubling of cumulative R&D inputs, the output metric will double \( r \) times.*

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134 There is technically a “knife edge” third possibility where software grows at a constant exponential rate, if software returns are *exactly* equal to the threshold. I’m setting this aside because it’s a knife edge result.
r > 1 means that doubling cumulative software inputs causes 2020-FLOP per FLOP to more than double.\textsuperscript{135} I discuss whether this is likely to happen here, considering estimates of r and the fact that r will diminish over time.

What are the implications of a software singularity for takeoff? In short, it would not guarantee a fast takeoff in every important metric, but it would make takeoff faster.

- **Make takeoff faster.** A software singularity would lead to very fast software progress as we approach AGI, significantly accelerating software growth. This means we cross the effective FLOP gap more quickly and AI capabilities improve more quickly immediately after AGI. This progress in AI capabilities wouldn’t be bottlenecked by the need to print new chips.

- **Increase the peak capabilities reached shortly after AGI.** If there’s a software singularity, AI software could rapidly grow by many OOMs and approach physical limits in the months after AGI, without needing to wait on the design and production of new AI chips. This has implications for how a small calendar lead in developing AGI could translate into a total capabilities advantage shortly after developing AGI.

- **No guarantee of fast takeoff.** During a software singularity, each doubling of software need only happen slightly faster than the previous doubling. In fact, only under extreme assumptions does each doubling happen twice as quickly as the last.\textsuperscript{136} This means that on a metric of takeoff speed in terms of the ratio between successive software doubling times, world 2 does not involve a fast takeoff even if there’s a software singularity. That said, a fairly rapid transition from world 1 to world 2 would be more likely to drive a fast takeoff if there’s a software singularity.

I’ve just discussed the internal dynamics of 2020-FLOP per FLOP in world 2 in their implications for takeoff speed; I analysed the dynamics of world 1 in section 4; now I describe a model of a gradual transition from world 1 to world 2.

\textsuperscript{135} Why is this the condition for software singularity? Suppose that you initially have 1000 AGIs doing software work. Let’s say it takes them 1 year to double cumulative software inputs. By this time, 2020-FLOP per FLOP has increased by a factor of $2^r$. (This follows straight from the definition of r.) If r > 1 then 2020-FLOP per FLOP has more than doubled, and so your research input has more than doubled to >2000 AGIs. How long will it take you to double cumulative inputs a second time? If your population of AGIs were still 1000, it would take you twice as long (each doubling of cumulative inputs takes twice as much effort as the previous doubling). But because you now have >2000 AGIs, it will take you less long and you’ll double cumulative inputs in less than a year. This means the growth rate of cumulative inputs is increasing. $g(output) = r \times g(cumulative\ inputs)$, so the growth rate of output is also increasing.

How does introducing a “stepping on toes” assumption change this analysis? Not much. Stepping on toes is expressed mathematically as $I = C^\lambda$, lambda < 1. In this case, the condition for software singularity becomes $r^\lambda > 1$. If we held our estimate of r fixed, then a software singularity would be less likely. However, consistency with the historical data requires us to raise our estimate of r to exactly compensate if we lower the value of lambda. This is because the historical data constrain $r^\lambda$ directly. For example, in section 4 I said the data suggested $r = 2.5$. But if I’d accounted for stepping on toes, I’d have instead said that the data suggests $r^\lambda = 2.5$. The implication of the historical data for the software singularity is unchanged. In both cases the singularity happens just if the quantity > 1, and the historical data suggest the quantity equals 2.5.

\textsuperscript{136} I analyse this dynamic more in appendix TODO.
Transition from world 1 to world 2

We can represent the annual inputs to software research mathematically in worlds 1 and 2.

In world 1, the annual inputs to software research are given by:

\[ I_s = L_s \quad (1) \]

where \( L_s \) is the number of human software workers.\(^{137} \)\(^{138} \)

In world 2, the annual inputs to software research are given by:

\[ I_s = C_s \quad (2) \]

where \( C_s \) is the quantity of 2020-FLOP used by AI systems for software R&D. (We will continue to use the same economic model to predict how the annual inputs \( I_s \) will affect the output metric, 2020-FLOP per FLOP.)

How can we model the move from world 1 to world 2? The best approach I know of here is the task based model.\(^{139} \) This model supposes that software R&D involves a large number of distinct cognitive tasks.\(^{140} \) Total R&D input depends on the inputs to each task.\(^{141} \)

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\(^{137} \) This is slightly different from section 4 where I measured software inputs in units of real $. In this section it will be simpler to talk in terms of # researchers, rather than real $. We can relate these two input metrics by estimating the annual rise in real salaries during this period, which I'll assume is 2%. Real $ inputs should grow 2% quicker than # researchers. For example, I estimated that real $ inputs to software would grow at 25%, and this would correspond to 23% growth in # researchers.

\(^{138} \) If we want to account for the “stepping on toes” effect we could instead write \( I = L^\lambda \). I won’t do this for simplicity of exposition, but will note in footnotes or appendices when the stepping on toes effect would meaningfully change the dynamics.

\(^{139} \) During my previous investigation into whether AI could drive explosive growth I didn’t come across anything more promising despite studying most mainstream growth models and most growth models of transformative Al (e.g. this review). This model also seems to be favoured by economists studying the economic implications of advanced Al, e.g. Aghion et al. (2017) and Hanson (2001). Some advantages of this model are: quantifying what % of the way from world 1 to world 2 we’ve travelled at each point in time and quantifying the effect of this on software R&D, allowing flexible incorporation of the degree of bottlenecks between different tasks (which I’ll use in section 6); being fairly intuitive to explain. With bottlenecks, the model looks like (see section 9.2) it can explain a good chunk of the growth of the last 150 years as resulting from automation. Thus we are using a model in which future Al automation is a continuation (and significant acceleration) of past automation (which for the first time ever leads to full automation).

\(^{140} \) In the model these tasks are fixed over time. However, in the version that incorporates bottlenecks (that I’ll introduce next section), the relative importance of these tasks does change over time. In particular, if we automate a task our output on that task increases and so the task becomes less important. The result is that the non-automated tasks grow in importance. This matches the recent trend of hard-to-automate sectors like education and healthcare growing as a fraction of GDP while automatable sectors like agriculture fall as a share of GDP. Also, the growing importance of non-automated tasks can represent the possibility that entirely new tasks are introduced that Al cannot (initially) perform. In our model, we’ll think of these new tasks as new applications of existing tasks that Al couldn’t perform.

\(^{141} \) This mathematical footnote is not needed to understand what follows. This section uses the Cobb Douglas version of the task-based model, which is simple and doesn’t include bottlenecks. In that model,
In world 1 humans do ~all the tasks and, when you do the maths of the task based model, this results in equation (1). In world 2 AIs do all the tasks and this results in equation (2). We model intermediate worlds as ones where AI performs a fraction $f$ of tasks, $0 < f < 1$. It turns out that, in the Cobb Douglas version of the task-based model (that excludes certain bottlenecks), total R&D input is given (up to an unimportant constant) by:

$$I_S = L_S^{(1-f)} \cdot C_S^f$$  \hspace{1cm} (3)

Notice that when $f = 0$ this becomes equation (1) and when $f = 1$ it becomes equation (2). As we continuously automate a greater fraction of tasks, the exponent on $C$ increases gradually from 0 to 1.

How should we model the process by which tasks are incrementally automated? There are two components here:

1. When will we develop AI that can perform various cognitive tasks?
2. When do you have enough runtime compute to actually automate various tasks?

When will we develop AI that can perform various cognitive tasks?

My proposal is to extend the Bio Anchors model for when we train AGI. Let’s say that we use Bio Anchors to predict that we’ll need $1e36$ 2020-FLOP to train AI that can automate 100% of the total R&D input depends on the input to each task as follows. You multiple the inputs of every task together to get the total R&D input. Mathematically, suppose there are $N$ tasks, and input to each task is given by $X_1, X_2, ..., X_N$. Then total R&D input $I = X_1 \cdot X_2 \cdot ... \cdot X_N$. The implication is that you want to spread your inputs evenly across the tasks, as a tiny input on any task could really let you down (and zero input on any task will mean zero total input).

Is it really true that human workers do all the tasks necessary for software development today? Perhaps conducting AI experiments is a crucial part of the process, and this “task” is already performed by computers. I discuss this in appendix TODO.

In the Full Takeoff Model, I actually assume that, even if AI experiments are not part of software development, initially a small percentage of software tasks are performed by 2020-FLOP. I am thinking here of the way in which software developers offload certain cognitive tasks to calculators and spreadsheets. These are only a small fraction of the relevant tasks because only a small fraction of the money invested in software development goes to buying the machines that do these types of tasks. (E.g. calculators are very cheap and use of google sheets are very cheap compared to a developer’s salary.) The assumption that a small percentage of software tasks, rather than 0 tasks, are initially performed by 2020-FLOP does not materially affect the results.

The model implies that an equal fraction of output is paid to each task in 2021. This means that the ‘fraction of tasks’ in the model matches my earlier definition of the “% of cognitive tasks”, where I weighted each task by the salary-weighted time spent on it in 2021. (I am assuming that all tasks performed by software workers are cognitive – as opposed to partly-physical tasks like “building a table” – and so can all ultimately be performed by Al.)

This section analyses the implications of the Cobb Douglas version of the task-based model because it is (relatively) simple to understand and tractable to analyse analytically. The Cobb Douglas version excludes certain bottlenecks, some of which are included in the CES version of the task-based model. I’ll flag when results from the Cobb Douglas version might not carry over to the CES version. I discuss the CES version, and bottlenecks more broadly, in section 6.
cognitive tasks (my definition of AGI). We can extend it to **estimate the 2020-FLOP needed to train AI that can perform x% of cognitive tasks, for various values of x**. For example, we might assume that you automate 50% tasks with 1e34 2020-FLOP and automate 20% of tasks with 1e31 2020-FLOP.

What evidence can we use to inform these assumptions?

- Bio Anchors, or perhaps other methods, can inform how much 2020-FLOP we think is needed to train AI that can perform 100% of tasks.
- My earlier discussion of the effective FLOP gap can then inform how much 2020-FLOP we think is needed to train AI that can perform 20% of tasks.
- The Full Takeoff Model (FTM) then uses a pretty hacky method to extrapolate to the training run needed to automate x% of tasks for any x.

The spread of these tasks in 2020-FLOP space, together with g(2020-FLOP), will dictate how quickly new tasks are automated. It is a hugely important and uncertain input to this framework, which strongly influences how suddenly we transition from world 1 where human workers are the key input to economic production to world 2 where AI is the key input.

When do you have enough runtime compute to actually automate various tasks?

As discussed in section 2, having AI that can perform a task is not sufficient to fully automate it. In addition, you must have enough runtime compute to actually replace the human workers that currently do the task.

To know whether we have enough runtime compute to fully automate a task, we need to know:

1. How many human workers are currently performing that task?
2. How much compute are we using to run AIs doing software R&D?
3. What are the runtime compute requirements for AIs to have the same output at the task as a human worker?
4. What one-off productivity gains do AIs have over humans?

Our Full Takeoff Model (FTM) makes assumptions about these quantities. The important high level points are that:

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145 In fact, to fully specify the model we’ll need to determine the 2020-FLOP needed for every value of x between 0 and 100, though the results are not sensitive to small variations.

146 Indeed, you must be able to run enough AIs doing the task that it becomes more profitable for human workers to perform a different task instead.

147 Of course, even this is not sufficient. You might have AI that can readily perform a task, and enough compute to affordably automate all instances of that task, but not actually automate it due to some other bottleneck like regulations, incumbents resisting automation, or just the minor effort involved in introducing the AI into the workflow. The Full Takeoff Model does not incorporate delays between getting enough training and runtime compute to fully automate a task and actually fully automating it; I discuss this weakness in section 10[TODO link]. (This is why the training compute threshold should be interpreted as the 2020-FLOP training requirement for AI to be able to “readily perform” x% of cognitive tasks.

148 Some additional details are included in this appendix.
1. FTM assumes 1.6 million people\textsuperscript{149} doing software R&D in 2022, which grows at 20% before “wake up” and 25% after “wake up”. Also workers’ time is split evenly over equally important non-automated tasks (so if there are 5 equally important non-automated tasks, workers spend 20% of their total time on each).

2. FTM tracks total global compute and, after “wake up”, the fraction of this used to run AIs doing software R&D rises very rapidly to \( \sim 10\% \). Why so high? After “wake up”, demand for AI software R&D will be high, and so a notable fraction of AIs will be assigned to it if they can be useful.

3. FTM assumes that the \textit{runtime} requirements for different tasks are spread out over multiple OOMs, just as the \textit{training} requirements.
   a. My central estimate has AGI at \( 1 \times 10^{17} \) 2020-FLOP/s and 20% of tasks at \( 1 \times 10^{15} \) 2020-FLOP/s.
   b. The spread of runtime requirements is smaller than the spread of training requirements for two reasons.
      i. A 10X increase in runtime compute typically corresponds to a 100X increase in training, e.g. for Chinchilla scaling.
      ii. Increasing the horizon length of training tasks will increase training compute but not runtime.

4. The FTM assumes \textit{\sim 60X one-time gains} for AGIs over humans doing R&D.

The software sector is relatively small and so lack of runtime FLOP only prevents task automation if AI training requirements are extremely low.

To summarise the above two subsections, a task is fully automated when we i) have done a big enough training run to develop AI that can perform the task, and ii) we have enough runtime compute for AIs to replace all humans at that task. I believe this is a natural way to integrate Bio Anchors with a task-based model of incremental automation.

Section 7 \textit{considers a model} in which having more runtime compute than you need to automate a task can compensate for not having enough training compute. The consequence is that software R&D can be fully automated \textit{much} earlier due to an abundance of runtime FLOP.

This simplistic model obviously omits many factors that in practice affect the automation of tasks; I’ll discuss this in section 8.

The dynamics during the transition from world 1 to world 2

During the transition, there is the following two qualitative feedback loops:

\textsuperscript{149} In fact, the true number of people doing AI software R&D is lower by 1-2 OOMs. My methodology was to multiply world population by an estimate the fraction of GWP spent on software R&D (0.02%); but in fact these workers’ salaries are much higher than average and per-person capital costs for this industry are also unusually high. It doesn’t matter much to the results as the bottleneck to automation is nearly always training compute rather than runtime compute.
(A) Better software \^150 \rightarrow more 2020-FLOP in largest training run \rightarrow more tasks automated \rightarrow more input to software R&D \rightarrow better software

(B) Better software \rightarrow more 2020-FLOP for running AIs \rightarrow more AIs per automated task \rightarrow more input to software R&D \rightarrow better software

Both loops increase inputs to software R&D; I’ve highlighted the differences in **bold**.

Qualitatively, the result of this feedback loop is that software R&D inputs and 2020-FLOP grow at increasingly fast rates – super exponential growth – as software tasks are automated.\^152

When does this super exponential growth become quantitatively significant?

In section 4 I guessed that the growth rate of real $ inputs to software R&D would be 25% after “wake up”. We can ask: **What fraction of tasks must be automated before the effect of AI on software inputs is larger than this?**

\^150 I use this interchangeably with “more 2020-FLOP per FLOP”.

\^151 [Link to chart.]

\^152 Equation (3) implies the growth rate of software inputs is given by \( g(I) = (1-f)g(L) + f g(C) \). We know that \( g(C) > g(L) \): 2020-FLOP grows much quicker than human inputs to software R&D. So as \( f \) increases, \( g(I) \) increases. If the growth rate of software **inputs** increases, so does the growth rate of software **output**: 2020-FLOP per FLOP. (This follows from the **SEG**). And if \( g(2020\text{-FLOP per FLOP}) \) increases then so does \( g(2020\text{-FLOP}) \), as long as \( g(\text{physical FLOP}) \) is not falling (in fact it will be rising). This establishes that both \( g(I) \) and \( g(2020\text{-FLOP}) \) increase as \( f \) increases. In short: \( f \) increases \( \rightarrow g(I) \) increases \( \rightarrow g(2020\text{-FLOP per FLOP}) \) increases \( \rightarrow g(2020\text{-FLOP}) \) increases. This argument could fail for two reasons. Firstly, in the CES version the importance of tasks done by AI falls over time and so we must automate tasks quickly enough to counteract this for the argument to go through. Secondly, if \( g(C) \) falls for some other reason (e.g. we stop ramping up the fraction of compute used for software R&D) then \( g(I) \) may fall and the argument is blocked.
Equation (3) implies that the growth rate of software inputs due to AI equals $f \times g(2020\text{-FLOP})$. If $g(2020\text{-FLOP}) = 100\%^{153}$, then this first exceeds 25% when $f = 0.25$. In other words, it is when roughly ~25% of cognitive tasks have been automated that rising AI inputs become more important to software R&D than rising human inputs. I’ll revisit this question in section 6 when we discuss bottlenecks, which will push towards a somewhat larger fraction.$^{154}$

**Summing up**

Incremental AI automation will increase $g(2020\text{-FLOP per FLOP})$ as we cross the effective FLOP gap. This effect is smaller than the effect of rising human inputs until AI has automated ~25% of cognitive tasks. By the time we reach AGI, 2020-FLOP per FLOP will be doubling in months or much less.

More generally, as we cross the effective FLOP gap, $g(2020\text{-FLOP per FLOP})$ depends on:

1. The returns to software R&D, quantified by the parameter $r$.
2. The rate at which inputs to software R&D grow. There are two sources of growth:
   a. Increasing number of people doing software R&D.
   b. Increasing fraction of tasks done by AI, and an increasing number of AIs doing each task.

This completes my discussion of the effect of incremental AI automation on 2020-FLOP per FLOP. The next section discusses the effect on FLOP/$.

**FLOP/$**

In section 4 I analysed how fast-rising AI investment might affect FLOP/$ after “wake up”. I guessed that inputs to hardware R&D might grow at 17%, eventually driving FLOP/$ to grow at ~88%.

This section extends that analysis by additionally considering the effect of incremental AI automation on FLOP/$. Like with software, we’ll see that $g(\text{FLOP}/\$)$ increases as more tasks are automated such that FLOP/$ may be doubling in months or quicker by the time we have AGI.

The analysis is very similar as for software, so I start by noting the points of similarity and difference.

$^{153}$ A ballpark figure from section 4.

$^{154}$ In fact, this analysis involved a few simplifications. Firstly, the bottleneck dynamic introduced in the next section will reduce the effect of AI automating 20% of tasks. Secondly, the effect of rising human inputs becomes less important as AI automates more tasks, which pushes in the other direction. Overall, I think the answer “$f = 0.2$” given here is too low, a more realistic answer is maybe $f = \sim 0.35$. [Maybe Jaime can investigate the relative contributions from AI automation vs rising human inputs for rho = -0.5?]
Comparing the effect of AI automation on hardware vs software R&D

The effects of AI automation on $g(\text{FLOP/}$ is similar to its effects on $g(2020-\text{FLOP per FLOP})$. Here are the key similarities in my analysis of each:

- We start in world 1, with humans doing 100% of the cognitive tasks needed for hardware R&D.
- We end up in world 2, where AIs do 100% of those cognitive tasks.
- There's a transition from world 1 to world 2 where the fraction of cognitive tasks done by AIs increases continuously from 0% to 100%. $g(\text{FLOP/}$ increases significantly during this transition.
  - During the transition, a task is automated when i) we’ve done a big enough training run that the resultant AI can perform the task, and ii) we have enough runtime compute to replace all humans doing that task.\textsuperscript{155}  \textsuperscript{156}

There are a few important changes:

- **Delays before innovation can boost AI capabilities.**
  - Software improvements can be rolled out immediately over all existing compute, increasing AI inputs to software R&D without delay. By contrast, there are significant lags between designing new chips and using the new chips to run AIs. At the least you need to manufacture new chips from an existing fab; you may also need to build new manufacturing equipment (e.g. for making chips of a new node size).
  - The FTM models the need to manufacture new chips, tracking both the stock of chips and the new chips produced each year. It also includes an optional lag between designing new chips and beginning to manufacture them. \textsuperscript{157} More.

- **Physical capital is needed for hardware R&D.** Hardware R&D sometimes requires experiments to test the behaviour of materials and new chip designs. To incorporate this, a fixed fraction $\alpha$ of tasks are performed by capital; the rest are cognitive tasks.\textsuperscript{157} My best guess is $\alpha = 0.3$.\textsuperscript{158}
  - The three equations from last section become:

\[ I_H = K_H^\alpha L_H^{(1-\alpha)} \] \textsuperscript{(1*) - humans do all cognitive tasks, world 1

\textsuperscript{155} FTM assumes 16 million people doing hardware R&D in 2022, which grows at 7% before “wake up” and 17% after “wake up”; people are split evenly over non-automated tasks. (16 million is too high by 1-2 OOMs; the reason is that we multiply world population by an estimate the fraction of GWP spent on software R&D (0.02%). It doesn’t matter much because the bottleneck to task automation is nearly always training compute rather than runtime compute.

\textsuperscript{156} The hardware R&D sector employs a relatively small number of people (though more than AI software) and so lack of runtime FLOP only prevents task automation if AI training requirements are low (e.g. AGI trained with <1e31 FLOP [TODO confirm]).

\textsuperscript{157} So this simplistic model assumes that all R&D tasks performed by labour are cognitive tasks, not requiring physical actuators. For labour tasks that aren’t cognitive, it is probably best within this framework to include them as tasks done by physical capital. More.

\textsuperscript{158} Alpha gives the fraction of R&D costs paid to physical capital (as opposed to cognitive labour) in 2021. This data suggests labour share is 65% and maybe as high as 90%, depending on whether you label certain subcategories are spent on labour vs capital. However, the data is for generic R&D rather than hardware R&D. It is also not possible to tell
$I_H = K_H^\alpha C_H^{(1-\alpha)}$ \quad (2*) - AI does all cognitive tasks, world 2

$I_H = K_H^\alpha L_H^{(1-\alpha)(1-f)} C_H^{(1-\alpha)f}$ \quad (3*) - AI does fraction f of cognitive tasks

where $K_H$, $L_H$, and $C_H$ gives the amount of physical capital, labour and 2020-FLOP used in hardware R&D; and $I_H$ gives the resultant real input to hardware R&D.

Having made this comparison with hardware vs software, I’ll briefly describe the dynamics affecting $g(\text{FLOP}/\$)$ in world 2 and in the transition from world 1 to world 2.

World 2

As with software, we consider two questions.

1. How long does the first doubling of hardware (FLOP/$) take in world 2?
2. How do the lengths of the software doublings change over time in world 2?

How long does the first doubling of hardware (FLOP/$) take?

The considerations influencing this are similar as for software. We forecast how many researcher-years will be needed to double hardware, and compare this to how many AGIs we’ll be able to run. If we’ll need 1 million researcher-years but we’ll have 2 million AGIs, the first doubling takes 6 months. The ballpark estimate here goes the same as for software, with the result that the first doubling could happen in months or less.

One additional complication here is that hardware R&D progress might be bottlenecked by limited physical capital. This could cause the first doubling to happen much more slowly.

How do the lengths of the software doublings change over time?

As with software, we can distinguish two scenario:

1. **Hardware singularity - quicker and quicker doublings.** If returns to hardware R&D exceed a certain threshold, the feedback loop is so powerful that there’s a “hardware only singularity”. The level of FLOP/$ grows faster and faster, theoretically going to infinity in finite time. This dynamic is initially curtailed by the difficulty of printing new chip designs fast enough to quickly match the current stock of hardware.

2. **Hardware fizzle - slower and slower doublings.** If returns to hardware R&D are below a certain threshold, FLOP/$ grows more and more slowly over time, assuming a fixed $ spend on FLOP.

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159 There is technically a “knife edge” third possibility where software grows at a constant exponential rate, if software returns are exactly equal to the threshold. I’m setting this aside because it’s a knife edge result.
Which scenario will obtain? The condition for **hardware singularity** is \((1 - \alpha)r > 1\). My best-guess values of \(\alpha=30\%\) and \(r = ~5\) imply it would happen comfortably. Though of course \(r\) will be lower by the time we reach AGI. The question is less important than for software because any hardware singularity would be slowed by delays printing chips, as mentioned above.\(^{160}\)

**Transition from world 1 to world 2**

Just like last time, there are two feedback loops at play during this transition:

\begin{enumerate}
\item[(A*)] Better hardware\(^{161}\) \(\rightarrow\) more 2020-FLOP **in largest training run** \(\rightarrow\) more tasks automated \(\rightarrow\) more input to hardware R&D \(\rightarrow\) better hardware
\item[(B*)] Better hardware \(\rightarrow\) more 2020-FLOP **for running AIs** \(\rightarrow\) more AIs per automated task \(\rightarrow\) more input to hardware R&D \(\rightarrow\) better hardware
\end{enumerate}

We can show these feedback loops alongside those for software:

\(^{160}\) Even if the conditions for software singularity don’t obtain and the conditions for hardware singularity don’t obtain, there can still be a joint hardware-and-software singularity if the combined returns are high enough. And even if this doesn’t obtain, I expect both FLOP/$ and software to eventually grow increasingly quickly due to GWP growth accelerating (which I discuss later).

\(^{161}\) I use this interchangeably with “more FLOP/$”.\n
It’s worth emphasising that the fast growth of 2020-FLOP is playing a dual role: bigger training runs which lead to greater automation and more runtime compute to run AIs doing tasks that have been automated.

AI automation becomes the dominant source of R&D input growth at about the same time as for software. In section 4 I guessed that human inputs to hardware R&D would rise at 17%. Equation (3*) implies that the growth rate of software inputs due to AI is given by the expression \((1 - \alpha) * f * g(2020\text{-FLOP})\). (Recall \(f\) is the fraction of cognitive tasks automated by AI.) If \(g(2020\text{-FLOP}) = 100\%\) (from section 4), then this first exceeds 17\% when \(f = 0.25\).\(^{163}\)

\(^{162}\) Link to diagram. There are of course human inputs to hardware and software R&D, but these aren’t represented explicitly in the diagram.

\(^{163}\) 100\%*0.7*0.25 = 17.5\%. Notice that \(f = 0.25\) is the same as the result for software R&D. There are in fact two differences between software and hardware here, which happen to roughly cancel out. First, I projected slower growth of human investments in hardware R&D than in software R&D. Second, I model physical capital (which AI can’t replace) as having an important role in hardware R&D but not in software R&D.
Summing up

Incremental AI automation will increase $g(\text{FLOP}/\$)$ as we cross the effective FLOP gap. This effect is smaller than the effect of rising human inputs until AI has automated ~25% of cognitive tasks (though the bottlenecks considered in the next section will increase this % somewhat).

More generally, as we cross the effective FLOP gap, $g(\text{FLOP}/\$)$ depends on:

1. The returns to hardware R&D, quantified by the parameter $r$.
2. The rate at which inputs to hardware R&D grow. There are two sources of growth:
   a. Increasing number of people and physical capital used in hardware R&D.
   b. Increasing fraction of cognitive tasks done by AI, and an increasing number of AIs doing each task.

This completes my discussion of the effect of incremental AI automation on FLOP/$$. The next section discusses the effect on $$ on FLOP.

$ on FLOP

In section 4 I analysed the impact of rising AI investments on $ on FLOP. I guessed that, after “wake up”, $ on FLOP would initially grow at a rate of ~97% as we ramp up the fraction of global FLOP used on the largest training run, and then at ~22% after this when we’re just expanding global chip production.

This section extends this analysis by incorporating the impact of incremental AI automation on $ on FLOP.

My approach here is to:

1. Use the same task-based model to forecast the effect of AI automation on Gross World Product (GWP) as I previously used to forecast its effect on software R&D and hardware R&D.
   a. This will tell us GWP in each year as we cross the effective FLOP gap and AI automates more and more cognitive tasks.
2. Assume that any acceleration in GWP growth accelerates growth in all economic sectors to the same degree. In particular, the increase in $g($ on FLOP globally) equals the increase in $g(\text{GWP})$.
   a. For example, suppose AI automation causes GWP growth to be 5% in some year rather than 3% - an additional 2% growth. Then I’ll assume that $g($ on FLOP globally) is 2% larger than I was previously assuming: 24% rather than 22%.
   b. This is a conservative assumption. AI automation of the economy will probably be disproportionately directed towards manufacturing more chips (more fabs, more chips per fab), given the large demand for FLOP that will exist.
   i. Importantly, the FTM does assume that AI automation will be disproportionately focussed on software and hardware R&D. Only on this
third component, $ on FLOP, am I making this conservative assumption.\textsuperscript{164}

c. For more detail see this appendix.

I’ll now say a bit more about #1, the effect of AI automation on GDP.

How I’m modelling the effect of AI automation on GWP

Just like I did for hardware and software R&D, I model incremental AI automation of the economy as a continuous transition between a world where cognitive tasks are performed by humans (world 1) to a world where they’re performed by AIs (world 2).

The equations here are the same as for hardware R&D, in that they include a constant fraction of tasks $\alpha$ performed by physical capital. GWP is given by:

\[
Y = \frac{K}{g} L^{(1-\alpha)} \quad (1') \text{ - humans do all cognitive tasks}
\]

\[
Y = \frac{K}{g} C^{(1-\alpha)} \quad (2') \text{ - AI does all cognitive tasks}
\]

\[
Y = \frac{K}{g} L^{(1-\alpha)(1-f)} C^{(1-\alpha)f} \quad (3') \text{ - AI does fraction } f \text{ of cognitive tasks}
\]

$Y$ gives GWP in each year;\textsuperscript{165} $K$, $L$, and $C$ give the amount of (physical) capital, human labour and 2020-FLOP used to produce goods and services (i.e. GDP) that year.

Equation (1’) is the standard \textit{Cobb Douglas} formula for GDP. Each time you double the quantity of labour ($L$), GDP ($Y$) doubles $\alpha$ times. And similarly, each time you double the quantity of capital ($K$), GDP ($Y$) doubles $\alpha$ times.

Equation (2’) simply replaces \textit{number of human workers} with 2020-FLOP, indicating that AI has fully automated the cognitive tasks previously done by humans.

Equation (3’) uses the task-based Cobb Douglas model to allow for a continuous transition between (1’) and (2’).

\textsuperscript{164} In other words, we are imagining that for each of the three components of 2020-FLOP ($\text{ on FLOP, }$ FLOP/$$, 2020-FLOP per FLOP) there is an equivalent sub-sector of the economy (chip manufacturing, hardware R&D, software R&D). FTM assumes AIs are disproportionately focussed on the latter two areas but not the first. While advanced AIs are heavily concentrated on improving software and chip design, they’re \textit{not} concentrated on building new fabs and expanding the capacity of existing fabs. Of course, the sharp division between hardware R&D and chip manufacture the model makes here is not entirely realistic; the two will often merge together in practice like when a new fab must be built to manufacture a new type of chip.

\textsuperscript{165} If growth is fast, $Y$ can increase significantly over the course of a single year. To account for situations like this, there is a more precise definition of $Y$: $Y$ gives the GWP that \textit{would be produced if output remained constant for 1 year}. Mathematically, $Y = ($ value of goods and services produced per second) $\times$ seconds in a year.
$\alpha$ turns out to be the fraction of GDP paid to capital, and $(1 - \alpha)$ is the share paid to cognitive labour. The actual share of GDP paid to labour in developed countries is ~0.65 and ~0.5 globally, but this includes physical labour as well as cognitive labour.\footnote{I'm using a simplistic model that \textit{completely ignores tasks that require physical labour}. Each task is either done by physical capital or it's a cognitive task that's initially performed by humans and later performed by disembodied AI. I discuss this in \textit{this appendix}.} I'll assume the share going to cognitive labour globally is 0.5, and so use $(1 - \alpha) = 0.5$.\footnote{Though 0.5 is too high for world GDP, I actually care more about the share of cognitive labour in the semiconductor industry: that's what's relevant for $\$ on FLOP$.

My thinking more generally is that cognitive labour is economically more valuable than physical labour by a wide margin, and so the share of GDP paid to cognitive labour should only be slightly lower than that paid to labour in total. Importantly, even jobs involving “manual labour” have a very significant component of cognitive labour to them. E.g. a plumber needs to figure out which changes to make and know how to make them; their distinctive skills mostly relate to these cognitive abilities rather than in their body’s ability to execute particular physical movements when instructed to do so by the brain. Empirical research could inform a better estimate of this parameter. You could look at the wages paid to various jobs in the US economy, and estimate the extent to which each job could in principle be done remotely (and so is purely cognitive) vs requires physical labour.} I'll assume the share going to cognitive labour globally is 0.5, and so use $(1 - \alpha) = 0.5$.

\begin{align*}
Y &= K^0.5 L^0.5 \quad (1') \\
Y &= K^0.5 C^0.5 \quad (2') \\
Y &= K^0.5 L^0.5(1-f) C^0.5f \quad (3')
\end{align*}

The conditions under which tasks are automated are the same as for the task-based models I’m using for software and hardware R&D. A task is automated when i) we’ve done a training run large enough that AI can perform the task,\footnote{As before, I adjust the Bio anchors estimate of 2020-FLOP training requirement for performing 100\% of cognitive tasks (AGI) to estimate the 2020-FLOP needed to train AI that performs only x\% of cognitive tasks for $0 < x < 100$. How widely distributed these thresholds are in FLOP space is very important, and informed by \textit{these considerations}.} and ii) we have enough runtime 2020-FLOP to run enoughs AIs to replace all humans at the task.\footnote{As before, I adjust the Bio anchors estimate of the 2020-FLOP/s needed to run AGI to estimate the 2020-FLOP/s needed to run AI that performs only x\% of cognitive tasks for $0 < x < 100$.}

There are the same feedback loops as before, as more $\$ on FLOP causes more tasks to be automated \textbf{and} more AIs to perform each task.

(A') Bigger GWP $\rightarrow$ more $\$ on FLOP $\rightarrow$ more 2020-FLOP in largest training run $\rightarrow$ more \textbf{tasks automated} $\rightarrow$ bigger GWP

(B') Bigger GWP $\rightarrow$ more $\$ on FLOP $\rightarrow$ more 2020-FLOP for running AIs $\rightarrow$ more AIs per \textbf{automated task} $\rightarrow$ bigger GWP

We can show these feedback loops alongside those for hardware and software.
Quantitative implications of the model

What does this model imply about how GWP growth changes over time? Equation (3') implies that the contribution to GWP growth from AI equals 0.5 * f * g(2020-FLOP). (Recall f is the fraction of cognitive tasks automated by AI; it increases over time.) Assuming g(2020-FLOP) = ~100%, this equals ~f * 50%.

With full automation (f = 1), this implies GWP growth is 63%, doubling roughly every year.\footnote{\textsuperscript{1}}

We get “explosive” GWP growth of >30% from AI when f = 0.6, i.e. with AI automates 60% of cognitive tasks. However, f will be higher once we take into account bottlenecks, as we’ll do in the next section.

Even so, if we quickly transition from a world where f < 0.1 to one where f > 0.8 then GWP could quickly go from doubling every ~20 years to doubling every ~2 years. In other words, there could be a fast takeoff according to the GWP doubling metric discussed above.

\footnote{The diagram doesn’t show inputs of human labour and physical capital to R&D and GWP. It only shows the AI inputs for simplicity.}

\footnote{By the time f = 1, g(2020-FLOP) is much higher and so this model will predict a faster GWP growth in practice. On the other hand, the model omits certain bottlenecks that will make growth slower.}
When does the effect of AI automation on $g(\text{on FLOP})$ become more significant than rising human investment? In section 4 I guessed that rising human investment would drive $g(\text{on FLOP}) = 22\%$. The contribution of AI to $g(\text{on FLOP})$ is the same its contribution to GWP growth: $f \times 50\%$. This exceeds $22\%$ when $f = 0.45$.

By contrast, we estimated AI automation would dominate human investment in software and hardware R&D when $f = 0.25$. So it seems like AI automation will dominate rising human investment for $g(2020-\text{FLOP per FLOP})$ and $g(\text{FLOP}/\$)$ before $g(\text{on FLOP})$.

I analyse why the Full Takeoff Model (FTM) can easily predict fast takeoff in GWP, in the context of what generic growth models say about takeoff speed, in this appendix.

I’m not modelling AI automation of generic R&D

The FTM does not include AI automating generic R&D and thereby causing a productivity explosion. This means I’m modelling the role of AI in producing goods and services but not in developing new technologies (other than those relating to software and hardware).

If I included this, it would increase the impact of AI automation on GWP and make takeoff faster. However, perhaps not that much faster: I believe the economic effects of AI automation via generic R&D will initially be smaller than its effects via goods and services.

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173 What’s driving the difference here? Firstly, I’ve assumed that capital is more important to GWP than to software or hardware R&D inputs. (Capital does 50\% of GWP tasks, 30\% hardware tasks, 0\% of software tasks.) AI automation doesn’t (directly) affect this capital component, so has smaller effects on GWP than on hardware and software R&D inputs. Secondly (and less importantly), I assume human inputs grow at 22\% for $\$ on FLOP vs only 17\% for hardware R&D. This means there’s a lower bar for AI automation to dominate inputs for hardware, compared with $\$ on FLOP.

174 Why? In short, because “doubling inputs to goods and services” immediately doubles GDP while “doubling inputs to R&D” only doubles the rate of tech progress, which takes many years to actually translate to a doubling of GDP.

In more detail: Imagine there are two sources of GWP growth: more production inputs and more R&D inputs. We want to compare using AI to increase production inputs via using AI to increase R&D inputs. There are two reasons to think the effect on GWP would be bigger and quicker via increasing production inputs. Firstly, data suggests that each doubling of R&D inputs causes less than 1 doubling of TFP; but in standard models doubling production inputs doubles GWP. This implies that doubling production inputs has a bigger effect on GWP. Secondly, to double GWP via production you only need to double annual production inputs. But via R&D what matters is cumulative R&D inputs, which are harder to double (even if your annual inputs instantaneously doubled, it would take a while for cumulative inputs to double). Combining these two points: to double GWP via production you need to double annual production inputs, but via R&D you need to more than double cumulative R&D inputs.

There are some good reasons to think increasing production inputs would have smaller effects than I’m claiming. i) Past a certain point people don’t want more of the same goods and services, they want new types of good; so increasing production inputs won’t help unless you’ve used R&D to invent new goods. ii) More technological, social and regulatory barriers to automating the provision of goods and services than to automating R&D. I discuss these further in TODO.
The takeoff model so far

The takeoff speeds model so far can be summarised as follows:

- We’re forecasting the calendar time to cross the effective FLOP gap.
  - effective FLOP gap = How much bigger does your training run need to be to automate 100% of cognitive tasks rather than just 20%?
- This depends on the size of the effective FLOP gap and on how quickly we’re able to increase the 2020-FLOP used in the largest training run.
- 2020-FLOP has three components such that:
  \[ g(2020\text{-FLOP}) = g(\text{on FLOP}) + g(\text{FLOP}/\text{$}) + g(2020\text{-FLOP per FLOP}) \]
- Section 4 analysed the effects of fast rising human investments on each of the three components. It can be thought of as estimating the effect of rising human labour and physical capital inputs on each component.\(^{176}\)

\(^{176}\) Section 4 estimated the growing input as measured in real $, rather than separately estimating the growth of labour and capital. The simplest assumption is that real inputs of both labour and capital are growing at this same rate.
This section analysed the effect of AI automation on each of the three components. As 2020-FLOP increases, we automate more tasks and have more AIs performing each task. These AI inputs are combined with human labour and capital inputs for each component.

[Link to diagram]
The growth of each component accelerates as we cross the effective FLOP gap and automate more tasks. By the time we have fully automated all the cognitive tasks for any given component, it doubles in a year or much less.

- We found that, for software and hardware R&D, the effects of AI automation are smaller than those of fast rising human investments (L and K) until ~25% of cognitive tasks have been automated. For $ on FLOP the threshold was ~45%.
- These thresholds will be somewhat higher after we take bottlenecks into account in the next section.
- Once we’ve reached these thresholds, the feedback loops in green and orange have become more significant in increasing g(2020-FLOP).

New metrics

The modelling introduced in this section, and the additional assumptions we must now make, allow us to calculate some other metrics of takeoff speed that were discussed in section 2.

- Successive GWP doubling times.
Equation (3’) calculates GWP based on the inputs of capital, (human) labour, and 2020-FLOP. As AI automates more tasks the importance of labour falls, that of 2020-FLOP rises, and GWP growth accelerates.

If we transition quickly from world 1 to world 2 GWP growth can quickly go from its current level (~3%) to much faster (>60%). The speed of this transition depends on the effective FLOP gap and on the average $g(2020\text{-FLOP})$ as we cross the gap.

The sensitivity analysis will report the largest ratio between successive GWP doubling times during this transition. Remember I’m calling ratios > 4 a “fast” takeoff, ratios <=2 a “slow” takeoff, and ratios between 2 and 4 a “medium” takeoff.

- **Time from “AI could readily automate x% of tasks” to “AI could readily automate y% of tasks”**.
  - To determine when tasks can be automated I make assumptions about i) the 2020-FLOP training requirements for each task and ii) the 2020-FLOP/s runtime requirements for each task.

- **How many AGIs can we run?**
  - The training and runtime requirements for the final task are the highest and can be loosely interpreted as the AGI training and runtime requirements.\(^{179}\)
  - The model calculates the largest training run in each timestep, so can calculate when we first train AGI
  - The model also calculates the total quantity of 2020-FLOP/s in each timestep, so can calculate how many AGIs we could run in each timestep.
  - This allows the model to calculate the first timestep in which we can train AGI and run X AGIs, for any X.
    - In fact it can calculate the number of AI that automates x% of tasks, for any x. AGI is the special case when x = 100.
  - I use this to calculate the metric **time from AI that could automate 20% of cognitive tasks to when we can run 10 billion AGIs.**
    - My startpoint of “automating 20% of cognitive tasks”, requires the training 2020-FLOP to train AI that can automate 20% of cognitive tasks and the runtime 2020-FLOP to run enough AIs to replace human workers at those tasks.

- **Cognitive output.**
  - As AI automates more cognitive tasks the model calculates total cognitive output in each timestep.
  - This is simply the output on tasks not performed by capital. The formula is:

\[
\text{Cognitive output} = L \frac{(1-f) C}{g} f
\]

\(^{179}\) Those requirements are sufficient to train an AI and run it to do any task as well as a human worker. In practice, AIs with lower requirements are used to perform most tasks in the model as this is more efficient. But you could in principle use a more expensive system to perform every task.
This notion embraces the complementarity between human and AI cognitive labour by tracking the output from both of them combined. It aims to side-step the complementarity between cognitive labour and physical capital in order to get a metric of AI capabilities that is independent of physical capital bottlenecks.

The notion also avoids leaning on any specific and arbitrary capability level like “AGI”; total cognitive output is the result of AI systems of varying levels of generality and capability, some of which may be tools and some of which may be agentic.

The units of cognitive output are remote human equivalents. More precisely, “How many remote human workers would be needed to generate the same economic value per day as we’re getting from the combined cognitive output of humans and AIs?”

I use this to calculate the metric: time from AIs being a 2X multiplier on human cog output to being a 10X multiplier.

Significant AI automation need not happen before AI risk

If you believe that the most likely path to AI causing existential risk is via accelerating economic growth, or having vastly superior cognitive capabilities, then there will be significant effects from AI automation before this happens. In this case, the dynamics discussed in this section are potentially extremely important as they precede x-risk.

On the other hand, if you think it’s likely existential risk comes from pre-AGI systems that perform some tasks excellently but cannot perform most tasks, the analysis of this section is much less relevant. AI won’t be sufficiently capable and general to accelerate software R&D, hardware R&D, or GWP before it poses x-risk.

My own view is that AI would probably need very significant degrees of autonomy in a very wide range of cognitive tasks to pose existential risk via power-seeking. For example, it would need to be able to perform ~all tasks in one or more broad areas like AI R&D, social manipulation, hacking and business/military strategy. And my guess is that this in turn would require AI to perform a large percentage of total cognitive tasks, probably >70%.

Summing up

This section analysed the effect of AI automation on the speed crossing the effective FLOP gap, g(2020-FLOP). I did this separately for each of the three components – g(2020-FLOP per

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180 So “cognitive output = 1 billion remote human workers” means “if we continued to produce cognitive output at the current rate for a year, then the total cognitive output produced would have the same value as that produced by 1 billion remote human workers working for 1 year”.

181 In particular, fully automating any of these high-level tasks requires many capabilities which will also help to fully automate or partially automate many other high-level tasks.
FLOP), g(FLOP/$) and g($ on FLOP) – using the same task-based model for each. Unsurprisingly, AI automation increases these growth rates significantly as we cross the effective FLOP gap. By the time AI has automated all cognitive tasks in a component, it doubles in months or faster.

When does the effect of AI automation become significant, compared to the rising human investments discussed in section 4? In the model used here, the answer depends on the component in question. For g(2020-FLOP per FLOP) and g(FLOP/$), i.e. for software and hardware progress, AI automation becomes significant when roughly ~25% of cognitive tasks have been automated; for g($ on FLOP) it was roughly when ~45% of cognitive tasks have been automated.

These AI automation dynamics are less relevant for takeoff speeds if you think AI will pose an existential risk before it automates a significant fraction of tasks.

Modelling AI automation required additional assumptions about the compute needed to train and run AIs that can perform x% of cognitive tasks for 0 < x < 100. These additional assumptions allow us to calculate the way in which the % of tasks performed by AI increases continuously over time.

The result model can calculate metrics of takeoff speed relating to GWP, the number of AIs, and the total cognitive output of AIs and humans.

What is the bottom line here for takeoff speeds? In this framework the values of takeoff speed metrics – e.g. how long from 20% automation to AGI – depend on the training and runtime requirements of pre-AGI systems, and how these combine with the rising human investments from section 4 and the bottlenecks described in the next section. Unfortunately, I’m not aware of a simple analytically tractable way to estimate them, and so this section did not make new central best-guess estimates of takeoff speed. Though I do take the numbers in section 4 to be very rough best guesses even accounting for AI automation. That’s because the numbers in section 4 are too high for when we start crossing the effective FLOP gap, and too low after AI automation kicks in.

The main body of the report continues in a new doc.

Appendices

Appendices to add:
- The shape of the task distribution over log(FLOP); maybe there’s a tail of tasks?
- Are we assuming “AGI is one big model” vs “lots of little models”?

182 Though I do take the numbers in section 4 to be very rough best guesses even accounting for AI automation. That’s because the numbers in section 4 are too high for when we start crossing the effective FLOP gap, and too low after AI automation kicks in.
Literature on brain size - IQ correlations

*Edit: a new meta analysis has been released, which comes down a little more conservative than I did here.*

My current guess is that a **10% more brain volume → 4.5 more IQ points.**

I skimmed three studies.

1. **Gignac & Bates (2017)** is a large recent meta analysis.
   a. Its headline figure is a correlation of **0.29** (95% CI = 0.24, 0.33) between brain volume and IQ.
   b. It found the result depended on the accuracy of the IQ measurement used. ‘Fair’, ‘good’, and ‘excellent’ measurements had correlations of 0.23, 0.32 and 0.39.
      i. It claims the adjustments it makes here are probably too small, as empirical measurements tend to be less reliable than normative samples used to rate measurement procedures.
   c. At a glance, it doesn’t seem to discuss confounders much. Health and education seem like possibilities. I don’t know how carefully the object level studies controlled for this.
   d. If the above two factors cancel (under-adjusting for mismeasurement of IQ and not including confounders) then the estimate of correlation due to causation is 0.3 - 0.4. (I’d guess this cancelling assumption leaves the correlation too high.)
   e. We can use the correlation to estimate that **a 10% bigger brain increases intelligence by 4.5 - 6 IQ points.**
      i. A correlation of z between X and Y means: increase X by 1 standard deviation → increase Y by z standard deviations. (This assumes the correlation is causal.)
      ii. I couldn’t see data about the standard deviation of brain sizes for the study participants; but this *seems to be ~10%* in the general population. So 1 standard deviation of brain size = 10% bigger brain.
      iii. So this study is saying a 10% bigger brain → 0.3 - 0.4 standard deviations of IQ, or 4.5 - 6 IQ points. (A standard deviation of IQ is 15 IQ points.)
   f. I’d guess this estimate is too high due to the seeming lack of effort to adjust for confounds, but I’m not confident about this.

2. **Nave et. al (2018)** is (I believe) the largest empirical study to date, bigger than all previous investigations combined (N = 13,608).
   a. The most relevant figure here is a correlation of **0.25.** (They get this after including various confounders and trying to adjust for mismeasurement of IQ.)
   b. This paper includes a few confounds (social deprivation, place of birth, height, genetics) and did other robustness checks.
   c. I expect they have under-adjusted for mismeasuring IQ, for the same reasons as Gignac & Bates (2017).
   d. We can use the correlation to estimate that **a 10% bigger brain increases intelligence by 4 IQ points.**
i. This is just as above, except that the standard deviation of brain volume in this study was 9.3%.\(^{183}\)

ii. The correlation means that a 9.3% larger brain $\rightarrow$ 0.25 standard deviations of IQ, or 3.75 IQ points. (Again assuming the measured correlation is causal.)

iii. So a 10% larger brain $\rightarrow$ 4 IQ points.\(^{184}\)

e. I don’t see a strong reason to think this is biased in either direction overall.

3. This 2019 sibling study (N = 1022) finds a correlation of 0.18 within families and 0.33 overall.

a. They of course control for family environment, which will include health and education.

b. Again, I expect they have under-adjusted for mismeasuring IQ. Indeed, they find strong evidence that their IQ tests are less reliable than they assume in their adjustment.\(^{185}\)

c. We can use the correlation to estimate that a 10% bigger brain increases intelligence by 3.5 IQ points.

i. This is just as above, except that the standard deviation of brain volume in this study was 8%.\(^{186}\)

ii. So a 10% larger brain $\rightarrow$ 3.5 IQ points.\(^{187}\)

d. I think this is too low, due to under-adjusting for the mismeasurement of IQ.

To summarise the above evidence on the effect of a 10% bigger brain:

- Gignac & Bates (2017): 4.5 - 6 IQ points. I’d guess this is too high, but I’m not sure. I don’t have a good understanding of this sprawling meta analysis.

- Nave et. al (2018): 4 IQ points. This doesn’t seem biased in either direction and is the biggest study out there.

- The sibling study: 3.5 IQ points. I think this estimate is too low, so I see this as easily consistent with the true effect being 4 - 5 IQ points.

Overall, I’d guess a 10% more brain volume $\rightarrow$ 4.5 more IQ points. I’ve adjusted slightly upwards from Nave et. al (2018) due to Gignac & Bates (2017). This corresponds to 1 standard deviation of brain size $\rightarrow$ 0.3 standard deviations of IQ.\(^{188}\)

\(^{183}\) See figure S3 in the supplementary material.

\(^{184}\) 3.75 * 10/9.3 = 4.03.

\(^{185}\) For reasons that are unclear, the correlation between Verbal and Performance observed in the MCTFR does not seem consistent with such high reliability. We nevertheless used a conservative value of 0.82 in calculating the disattenuated associations; the assumption of a lower value would lead to larger apparent effects.

\(^{186}\) Figure S1 finds brain volume mean (standard deviation) is 1270 (101) for males, and 1100 (88) for females. 1270/101 = ~1100/88 = ~ 8%.

\(^{187}\) 10% bigger brain $\rightarrow$ 0.18 * 10/8 = 0.23 standard deviations of IQ = 3.45 IQ points.

\(^{188}\) Assuming 1 standard deviation is a 10% increase in brain size and a 15 point increase in IQ.
Ramp up will be bottlenecked by supply of FLOP

In this regime of fast rising investment, I expect the primary bottleneck for investment is not going to be willingness to pay, but instead supply constraints.

For example, if someone wanted to spend $100 billion on AI chips today, they simply couldn’t (NVIDIA data center revenue in 2022 was ~$10b, and they’re a large fraction of the AI chip market). If the person insisted on spending that much, they’d be forced to buy non-AI chips that are much less well suited for AI. In this example, the bottleneck on “get a quantity of AI chips that would be worth $100 billion at current prices” is not willingness to pay but instead how quickly chip manufacturers can scale up production of AI chips. Large willingness to pay can expedite this process, but only to some extent.

Similarly, if someone wanted to spend $100 billion on AI software researchers today, the key bottleneck would be talent availability. If they wanted to hire good quality people, they’d be limited by the time it takes to attract and train good people to grow a small field. In this example, the bottleneck to “get a quantity of AI software workers that would cost $100 billion at today’s prices” is how quickly you can attract and train high quality talent.

So I analyse how quickly we can ramp-up AI investments by focussing primarily on the supply side.\footnote{I don’t think supply places a strict bottleneck on annual AI investment. If there’s higher willingness to pay on the margin, that will somewhat increase real AI investment by inducing more people to abandon otherwise lucrative activities. So demand does make a difference on the margin. But, past a certain point, that marginal difference is small and you approach hard limits in terms of (e.g.) the time it takes to find and train additional people, and the limited number of people who have the expertise to deliver that training.}

I ignore the rising price of inputs to AI development after wake up

One factor I (try to) put to one side is the likelihood that the cost of inputs to AI investment will rise significantly as demand far outstrips supply. If the actual number of AI software workers remains constant, but their salaries have all doubled, I don’t want to say that software investment has doubled. Instead, I would say that real software investment has stayed constant. The numbers in this report should all be interpreted in this vein as referring to the growth of real inputs to AI, measured in the number of quality-adjusted workers, physical capital and computer chips. In this way, I (try to) sidestep the way in which high demand will drive up the price of real AI investments.\footnote{I feel confused about whether this move will lead to unrealistic predictions about the things I care about. E.g. I will end up talking about “FLOP/" numbers that, because I’m ignoring the effect of high demand on prices, are predictably too low. But what I ultimately care about is the total FLOP available in the world, not the amount that is paid for; I’m only using “FLOP/" as a measure of hardware progress. The question is whether my forecast of the available FLOP is distorted by not explicitly modeling this factor. I’ll forecast “$ on FLOP” numbers that are predictably too low for the same reason, and again it’s unclear how much this matters.}
Bold assumption to make the analysis somewhat tractable

One thing I’ll need to forecast is the growth of FLOP produced each year globally.\textsuperscript{191} By analogy with section 3, I calculate this as.\textsuperscript{192}

\[
\text{FLOP per year} = \text{$ on FLOP per year} \times \text{FLOP/$}
\]

I forecast each of the two components by mapping them to two sources of growth in FLOP production. The two sources are:

1. **More chips.** Increases in the *number of chips produced per year*. E.g. more fabs, more production lines within each fab.
2. **Better chips.** Increases in *FLOP per chip*. E.g. smaller node sizes, specialised chip designs.

In reality, I suspect these two sources can’t always be cleanly separated.\textsuperscript{193}

My bold assumption is that “more chips” corresponds exactly to more $ on FLOP, and that “better chips” corresponds exactly to more FLOP/$. In particular, I assume \(g($ on FLOP per year) = g(\text{number of chips produced per year})\), and that \(g(\text{FLOP/$}) = g(\text{FLOP per chip})\).\textsuperscript{194}

Then my strategy is to:

- Forecast \(g($ on FLOP per year)\) via forecasting how quickly we will expand chip production after “wake up”.
- Forecast \(g(\text{FLOP/$})\) by assuming that hardware R&D has driven historical growth in FLOP/$, and forecasting inputs to hardware R&D after “wake up”.

Perhaps the right thing to do is to replace “FLOP/$” with “FLOP/s per chip” and replace “$ on FLOP” with “number of chips”, to avoid the reference to “$”.\textsuperscript{195}

\textsuperscript{191} What do I mean by “total FLOP produced each year”? Take all the chips produced over the course of one year, run them all non-stop for one year, and ask: *How many FLOP did you do?* This is what I mean. A more precise statement would be “the annual FLOP capacity of 1 year’s chip production”. I use this unit so we can talk about FLOP rather than FLOP/s. This is useful because I’m ultimately concerned with how many FLOP we have available for the largest training run in each year, and only indirectly concerned with FLOP/s.

\textsuperscript{192} In section 3 I calculated FLOP for the largest training run = $ on FLOP for the largest training run * FLOP/$. (I’m putting aside software progress for now.)

\textsuperscript{193} Imagine we build a new fab with smaller node size, and compared its FLOP production to an old fab. We ask: *Is the new fab’s greater FLOP production due to better chips, or due to expanded production?* The new node size may use a completely different kind of chip that doesn’t map cleanly to the old chip. As a result, it may be ambiguous whether the new fab has more chips, relative to the old fab. So it’s ambiguous to what extent the greater FLOP production of the new fab comes from more chips vs better chips.

\textsuperscript{194} I.e. I assume that the price of chips is constant. People create better chips so that they can sell more chips at the same price, not to increase the price per chip. I think this assumption is more accurate over long timescales than short timescales. Over short timescales, you might be able to sell better chips for more. But in the long run, the price of the most recent cutting edge chips may be constant at ~$10,000 per chip.
<table>
<thead>
<tr>
<th>Source of growth</th>
<th>Equivalent quantity in takeoff framework</th>
<th>Cause of growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>More chips</td>
<td>g($ on FLOP per year)</td>
<td>Build more fabs and bigger fabs</td>
</tr>
<tr>
<td>Better chips</td>
<td>g(FLOP/$)</td>
<td>Hardware R&amp;D</td>
</tr>
</tbody>
</table>

I discuss why my bold assumption might be wrong, and how a more realistic assumption might change the results, in the following footnote.¹⁹⁵ This part of the framework feels conceptually confused, and I’d welcome suggestions for improvement.¹⁹⁶

### Details about assumptions of the Full Takeoff Model

*This appendix gives some additional details on assumptions made by the Full Takeoff Model (FTM). For additional information you could:*

- See the FTM’s behaviour for your chosen deterministic inputs [here](#), including justifications for my preferred values.
- Inspect the functionality of [this old spreadsheet](#) version of the FTM.
- Ask Epoch for the most up to date python code.
- Read this [concise mathematical description of the FTM](#) (courtesy of Epoch).

### Accounting for the “stepping on toes” effect when estimating the returns to hardware R&D

Suppose you invest $X in R&D. If there’s a stepping on toes effect, then the effective R&D input is only $X^\lambda$, $\lambda < 1$. Some effort is duplicated (or otherwise wasted due to the difficulty of parallelising R&D effort). So doubling investment only increases effective inputs by $2^\lambda$.

¹⁹⁵ I suspect my assumption gives too much credit to “more chips”, and not enough credit to “better chips”, in explaining the historical growth of FLOP production. I assume that all the increase in [$ on FLOP] is due to “more chips”. But the price of chips has probably increased over time, so that some of the increase is due to “better chips”. How would giving more credit to “better chips” change the results? Firstly, it would mean giving historical hardware R&D more credit for the growth in FLOP production, and so increase my forecast of how quickly hardware R&D will increase FLOP production after “wake up”. Secondly, it would slightly lower my estimate of how quickly we’ll be able to expand chip production after “wake up”. I’m not sure how these effects would net out; my guess is that the first would be larger and that the net effect would be fairly small.

I assume that the only reason why [$ on FLOP] increases is because of more chips; but in reality I’d guess that it also increases due to better chips. People pay more for SOTA chips over time. This means that

¹⁹⁶ Repeating from an earlier footnote, perhaps I should replace “FLOP/$” with “FLOP per chip” and replace “$ on FLOP” with “number of chips”, eliminating the reference to “$” entirely.
How does this affect our empirical estimate of the returns to hardware R&D?

Let $g_I$ be the growth of R&D inputs, $g_O$ be the growth of the relevant R&D output metric, $\lambda$ be the stepping on toes parameter and $r$ be the returns to hardware. For hardware R&D increasing FLOP/$, we have historical observations of $g_I$ and $g_O$ and must infer $\lambda$ and $r$ from the data. With the semi-endogenous growth model, if $g_I$ grows at a constant rate, the equation linking these quantities is:

$$\frac{g_O}{g_I} = \lambda \times r$$

(1)

Here’s what happens in the main text. Let $g_{I\_h}$ and $g_{O\_h}$ be the observed historical growth rates of R&D inputs and FLOP/$. In the main text I assume $\lambda = 1$ and then use (1) to infer $r = \frac{g_{O\_h}}{g_{I\_h}}$. I then make a hypothesis about the future growth of inputs after “wake up” $g_{I\_f}$, and infer future growth of the FLOP/$ $g_{O\_f}$. Mathematically, this is:

$$g_{O\_f} = g_{I\_f} \times r = g_{I\_f} \times \frac{g_{O\_h}}{g_{I\_h}}$$

$g_{O\_f}$ and $g_{O\_h}$ give historically observed growth of hardware inputs and FLOP/$; g_{I\_f}$ and $g_{O\_f}$ are forecasts of the same quantities after “wake up”.

If instead I’d assumed some stepping on toes then I’d used (1) to infer $r = \frac{g_{O\_h}}{(g_{I\_h} \times \lambda)}$. My estimate of $r$ would increase by a factor $1 / \lambda$, as the growth of effective inputs have increased more slowly due to stepping on toes. Then I’d have made the same forecast about growth of inputs after “wake up” and inferred future growth of FLOP/$$ as follows:

$$g_{O\_f} = g_{I\_f} \times \lambda \times r = g_{I\_f} \times \lambda \times \left( \frac{g_{O\_h}}{g_{I\_h} \times \lambda} \right) = g_{I\_f} \times \frac{g_{O\_h}}{g_{I\_h}}$$

$g_{O\_f}$ is exactly the same. My estimate of $r$ is higher in a way that exactly offsets stepping on toes. So stepping on toes doesn’t affect the predicted growth of FLOP/$$ after “wake up”.

There is one significant caveat. Equation (1) assumes annual inputs are growing at a constant rate. If annual inputs start growing more quickly than they used to – like they will after “wake up” – things are more complex. In this case, a stepping on toes dynamic ($\lambda < 1$) will increase the lag between the faster growth of annual inputs and the faster growth of the output. You can see this dynamic play out in this sheet. So the stepping on toes effect increases the lag between “faster growing hardware R&D inputs” and “faster growing FLOP/$” and so makes takeoff slower.

The diminishing returns to hardware and software become steeper over time; ideas become increasingly hard to find

By the time we reach physical limits of hardware, further progress is impossible. This corresponds to $r = 0$ (each doubling of cumulative R&D inputs causes 0 doublings of FLOP/$$).
The FTM assumes \( r \) decreases towards 0 by a constant amount each OOM of FLOP/$ increase between now and the physical limit. E.g. if we’ve 6 OOMs from the physical limit and currently \( r = 2 \) then by the time FLOP/$ has increased by 3 OOMs, the FTM assumes \( r = 1 \).

This dynamic can capture the expectation that hardware returns should trend back to the average R&D returns across all sectors of the economy.\(^{197}\) As long as the physical limits are assumed to be at or above 1e25 FLOP/$, introducing physical limits in this way doesn’t significantly affect the results.

And the same model is used for software.

You can see the assumptions about these physical limits, and justifications, in the “additional parameters” tab here.

### Full derivation of the equation for hardware R&D

It’s the standard semi-endogenous equation, with a "stepping on toes" effect, and with two complications.

Here’s a derivation of the equation ignoring "stepping on toes" and the additional complications.

---

\(^{197}\) *Are ideas getting harder to find* estimates the average returns to the overall economy to be \( r = \frac{1}{3} \), much lower than the returns for hardware.
\[
\dot{A} = \delta r(t) A^\phi \quad \text{(Assumption 1)}
\]
\[
\dot{A}/A^\phi = \delta r(t)
\]
\[
\int_{-\infty}^{t} \dot{A}/A^\phi \, dt = \delta \int_{-\infty}^{t} r(t) \, dt
\]

Let \( R(t) \) be the total researcher-years by time \( t \):
\[
R(t) \equiv \int_{-\infty}^{t} r(t) \, dt
\]
\[
\int_{-\infty}^{t} \dot{A}/A^\phi \, dt = \delta R(t) \, dt
\]
\[
A^{1-\phi}/(1-\phi) = \delta R(t) \quad \text{Assumption 2 ensures } \varphi \neq 1
\]
\[
A^{1-\phi} = (1-\phi)\delta R(t)
\]
\[
A = (1-\phi)\delta R(t)^{1/(1-\phi)}
\]

\[
A(t) = kR(t)^m, \text{ with positive constants } k = [(1-\phi)\delta]^{1/(1-\phi)} \text{ and } m = 1/(1-\phi)
\]

Notice this implies that \( g(A) = m \ast g(R) \). In the report I use different variable names, so let's stick with those of the report: \( g(O) = r \ast g(I) \).

Now to include stepping on toes in this set-up I alter the semi-endogenous equation in the standard way by adding a parameter \( \lambda \): \( A = \delta \ast r^\lambda \ast A^\phi \).

I then define \( R(t) = \int_{-\infty}^{t} r(t)^\lambda \, dt \)

This (I claim) doesn’t change the result of the above derivation. We again get \( A = \text{constant} \ast R^\lambda(1/(1-\phi)) = \text{constant} \ast R^\lambda \cdot r \). So \( g(A) = r \ast g(R) \).

But my new definition of \( R \) implies \( g(R) = \lambda \ast g(r) \). So \( g(A) = r \ast g(R) = r \ast \lambda \ast g(r) \).

In the report's notation: \( g(O) = r \ast \lambda \ast g(I) \).

Then two complications are added:

1. For hardware R&D (but not software R&D) I replace \( r^\lambda \ast \lambda \ast g(r) \) with CES\((r^\lambda \ast \lambda \ast g(r), K^\lambda \ast \lambda) \) where \( K \) is the physical capital used for R&D and \( r \) continues to be the number of researchers. This allows for physical capital to play a role in R&D, and potentially allows for physical capital to bottleneck R&D progress.
2. The returns to R&D r decreases towards 0 as we approach physical limits (more).

For a full implementation reach out to Epoch or check out this (messy!) sheet.

Aggregating one-time productivity gains from different sources

My approach is to quantify the individual effects mentioned, combine the relevant ones together, and maybe do a final adjustment based on my “gut”.

Productivity gains from different sources:

- Faster serial speed
  - Paul and Carl think a 1000X speed-up is possible
  - E.g. 1 AGI running for 1000 subjective years rather than 1000 humans working for 1 year each
  - I’m calling this ~10X
- No leisure / sleep: 3X (people spend 8 hours a day working)
- Better motivation: 2X
- Average vs top productivity
  - Among humans, average vs top productivity is >100X (global average income is ~$10k, most productive people can earn >$1m)
  - But all AGIs are as productive as the most productive AGI
  - So this naively gives AGI a gain of ~100X

Multiplying these gains together gives 10 * 3 * 100 * 2 = 6,000.

**One-off gains for AGI in R&D.** I will exclude “average vs top productivity” as researchers are mostly close to the top global average productivity. That leaves me on 10 * 3 * 2 = 60X.

**One-off gains for AGI in goods production.** I don’t think serial speed will apply very much in goods production. I also feel pretty suspicious of the “average vs top productivity” figure, in particular that AGIs could increase everyone’s productivity by 100X despite lacking the context of their jobs. This leaves me 3 * 2 = 6X.

In the FTM these assumptions are combined with a starting estimate of the AGI’s runtime compute of 1e17 FLOP/s using 2020 algorithms. See rows 5 and 7 here.

**Modelling $ on FLOP**

Roughly speaking, I assume:

$ on FLOP for training run = GWP * fraction of GWP spent on FLOP * fraction of FLOP on largest training run

FLOP for a training run = $ on FLOP for training run * FLOP/$

---

198 It turns out that you can derive the size of this effect from the size of the “stepping on toes” effect. If you quantify that effect in the usual way with lambda < 1, then a 1000X speed up increases productivity by 1000^*(1-lambda). 10X gain corresponds to lambda = ⅓, which is roughly my best guess for lambda.
The quantities are forecast as follows:
- GWP: use the task-based model for how AI automation affects GWP
- Fraction of GWP on FLOP: section 4 analyzes of growth of $ on FLOP globally
  - I guessed $ on FLOP globally might grow at ~22% after “wake up”
  - GWP growth is 3%, so this implies that the fraction of GWP on FLOP will grow at ~19%.
  - So my central estimate is ~19%.
- FLOP/$: grows due to hardware R&D. Section 4 analyses the effect of human inputs; section 5 analyzes the effect of AI automation.
- Fraction of FLOP on largest training run: analyzed in section 4.

**Annual production vs stock**
The above equation ignores that we can use chips bought in previous years in training runs. In fact the equations used in the FTM are:

\[
\text{global FLOP year } y+1 = \text{global FLOP year } y + \text{GWP} \times \text{fraction of GWP on FLOP} \times \text{FLOP/$}
\]

FLOP for training run = global FLOP * fraction of FLOP used on largest training run

In essence, the above equation pretended that we produce all our chips from scratch each time-step, while these ones allow us to accumulate a stock of chips over time. Each year’s production simply adds a little to that stock.

I do not explicitly distinguish between FLOP used for AI and other FLOP. I think this is a weakness of the FTM as it stands, which I explain more in this doc.

**FTM assumes no tasks are done by physical labour**
- The better model would have:
  - Some tasks done by physical labour, some by cognitive labour, some by physical capital
  - One process whereby AI automates cognitive labour.
  - A second process whereby robotics automates physical labour.
- But my model only includes the first process: AI automating cognitive labour. I ignore physical labour.

**Calculating the training requirements for “AI can readily perform x% of cognitive tasks”**
The Full Takeoff Model (FTM) makes assumptions about the training requirements for “AI that can readily perform x% of cognitive tasks” for all x. In other words, it makes an assumption about the full shape of the following graph:
The FTM calculates the full shape of the curve mechanically from just one input, the effective FLOP gap from 20% of tasks to 100%. The method, described mathematically here, creates a curve shaped like that in the picture above. In particular:

- Each additional OOM of training unlocks more tasks than the last.
  - This seems very likely as recently each OOM of training has seemingly unlocked very few tasks. I’d expect a gradual transition with each OOM unlocking more tasks than the last.
- The effective FLOP gap from ~1% to 20% is half the effective FLOP gap from 20% to 100%.
  - My best guess would actually be that ~1% to 20% is roughly as big as 20% to 100%, or bigger.
  - But if we used that assumption, we’d have less flexibility in specifying the 20%-100% effective FLOP gap. E.g. suppose we wanted to say AGI is 1e30 FLOP and the effective FLOP gap is 4 OOMs. Then our assumption would imply that ~1% of tasks required training of ~1e22 FLOP. But we’ve already seen training runs of 3e24 FLOP, implying that today’s systems can readily perform >>1% of cognitive tasks. The model would predict that AI could readily add >>$500 trillion/year to GDP, in contrast to observed AI revenues.

---

199 Annual revenues from AI are estimated at ~$10 - 100b. (E.g. here, here, here, here; I don’t know how reliable these estimates are, or even their methodologies.) But performing even 1% of tasks would be worth ~$500b because about ~$50tr is paid in wages each year globally.

200 Reminder: the phrase “readily” here indicates that i) it would be profitable for organisations to do the engineering and workflow adjustments necessary for AI to perform the task in practice, and ii) they could make these adjustments within 1 year if they made this one of their priorities.
○ Our actual assumption allows us to specify scenarios with fairly large effective FLOP gaps without this implied inconsistency. This is a reason to expect the actual effective FLOP gap to be smaller, than the maximum effective FLOP gap allowed by the model.

Different assumptions about the exact shape of the curve would change the bottom line somewhat. And perhaps some useful insight could come from thinking more carefully about implications of different curves, and testing out different possibilities (e.g. a log-normal distribution). But I suspect most of the ‘action’ is in the single scalar parameter I’ve pulled out – the effective FLOP gap – that describes how spread-out in FLOP space different tasks are.

How does physical capital changes over time in the FTM?

- Cognitive output is grows fast due to AI automation.
- Initially, this increases GWP a lot because cognitive tasks are important to GWP (they’re paid a high fraction GWP).
- More GWP → more reinvestment → faster growth of physical capital.
- But after a while, the abundance of cognitive tasks reduces their importance to GWP (they’re paid a much lower share of GWP). GWP is bottlenecked by physical capital. Further cognitive output growth doesn’t affect GWP much at this point. So the reinvestment in physical capital stops rising: ~constant GWP → ~constant reinvestment → ~constant growth of physical capital.
- Eventually physical capital grows faster and faster because of tech progress
  - Higher TFP allows you to accumulate capital more quickly.
  - This takes unrealistically long in the FTM.
  - The reason it takes so long is that the FTM assumes TFP grows exogenously rather than modelling AI automation’s effect on generic R&D; it takes many decades before physical capital is doubling every year.
  - In fact I expect it would take much less long that for billions of AGIs to design robot-factories that collectively self-replicate in a year.

How much does AI automation accelerate our progress through the effective FLOP gap overall?

We re-ran all the scenario analyses from this section, excluding the speed-up effect of AI automation. This allowed us to compare the takeoff speed with and without AI automation.

<table>
<thead>
<tr>
<th></th>
<th>Takeoff speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time from AI that could</td>
<td>Time from AI</td>
</tr>
<tr>
<td>readily</td>
<td>could readily</td>
</tr>
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</tr>
<tr>
<td>readily</td>
<td>could readily</td>
</tr>
</tbody>
</table>
automate 20% economic tasks to AI that could readily automate 100%. automate 20% R&D tasks to AI that could readily automate 100%.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Including AI automation</th>
<th>Excluding AI automation</th>
<th>Including AI automation</th>
<th>Excluding AI automation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short, best guess</td>
<td>3.7</td>
<td>16.5</td>
<td>4.4</td>
<td>8.8</td>
</tr>
<tr>
<td>Medium, best guess</td>
<td>5.0</td>
<td>14.0</td>
<td>5.5</td>
<td>12.5</td>
</tr>
<tr>
<td>Long, best guess</td>
<td>15.7</td>
<td>63.6</td>
<td>18.0</td>
<td>56.8</td>
</tr>
<tr>
<td>Medium, aggressive</td>
<td>0.4</td>
<td>0.9</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>Medium, conservative</td>
<td>27.0</td>
<td>46.5</td>
<td>32.3</td>
<td>49.4</td>
</tr>
</tbody>
</table>

Overall, it looks like AI automation increases our speed crossing the effective FLOP by ~2.5X.

Am I assuming AGI will take the form of one unified system, or many narrow systems working together?

Early writing about AGI tended to assume that it would take the form of one system with general capabilities. Others have since suggested that AGI could instead consist of many individually narrow AIs which work together to collectively have general capabilities.

When developing this framework, I tried not to take a strong stance on this question. Ultimately, the mathematical form of the Full Takeoff Model (FTM) fits best with the latter view. The FTM has dozens of different tasks, which each have different training and runtime requirements. The most natural interpretation is that different AIs perform each task.

If you want to make the FTM consistent with the “one general system” interpretation of AGI, you could say that there’s one AI that i) learns to do more tasks as we increase the size of our training runs, and ii) can match human performance at some tasks with OOMs less compute than other tasks. But this interpretation seems less natural to me.\(^{201}\)

But then why do you talk about the “number of AGIs we could run” as if AGI was a unified system?

\(^{201}\) In particular, how is AGI able to match human performance on some tasks with OOMs less compute than others? Perhaps it does them in much less time, but the more natural explanation is that it delegates those tasks to a smaller model, in which case we are back to having many different AIs.
To calculate the number of AGIs we could run, I divide the globally available FLOP/s by the FLOP/s required to run the AI that performs the most compute-intensive task (task with the highest runtime compute requirements).

This calculation would be accurate if one unified system can use this amount of FLOP/s to match human performance on any task, and can’t perform any tasks with less FLOP/s.

This calculation is conservative by the lights of the FTM, in that it underestimates the number of remote human workers whose output we could match by running AIs. This is because (according to the FTM) many tasks can be done with much less runtime compute than the most compute-intensive task.

But your assumption that a massive training run will be needed to train AGI implies AGI will be one unified system.

All that is strictly-speaking implied is that a massive training run will be needed to automate the hardest-to-automate tasks. I expect many tasks will be performed by smaller and more specialised AIs than this. That said, I do expect the AI trained in that massive training run to have fairly general capabilities.

Value-weighted cognitive tasks

Throughout the report, whenever I refer to the % of cognitive tasks – or the fraction of cognitive tasks – I am using a particular method for weighting different tasks.

Roughly and intuitively, each task is weighted by how important it is to the economy in 2020. A task that many people perform, and are paid lots of money to perform, has more weight than a task performed by fewer people on lower wages. More precisely, a task’s weight is proportional to the total $ that people earn while performing the task.\(^{202}\)

Even more precisely, a task’s weight is given by the elasticity of GDP to that task in 2020. If you performed 1% more of that task, how much would that increase GDP? If the answer is “GDP would increase by x%”, then the task’s weight is x. The task-based models I’m using assume that performing 1% more of every task (both cognitive and non-cognitive tasks) would increase GDP by 1%, so the total weight of all tasks equals 1. If economic inputs are allocated efficiently, this definition should match the one relating to wages.\(^{203}\)

The weights are pinned to a particular year (in my case 2020 for convenience) because the relative economic importance of tasks changes over time. In particular, tasks that are automated and so can be performed in higher volumes and quality tend to become less economically

\(^{202}\) For each person, this is given by the time they spend on the task multiplied by their hourly wage.

\(^{203}\) For tasks performed by physical capital, a task’s weight should equal the total amount paid to rent capital to perform the task.
important over time (e.g. producing food). Tasks that cannot be automated or made more productive tend to become more important (e.g. those in healthcare and education).

This same effect happens in the Full Takeoff Model (FTM) as AI automates cognitive tasks. Tasks that AI can perform in great quantities become less economically important – their weight decreases – ones still performed by humans become more important. The weights change over time, and so I pin my weights to 2020 when referring to the % of cognitive tasks.

An analogous definition of task weights applies to R&D as to the broader economy. As before, the precise definition ties a task’s weight to how much performing more of the task would increase R&D output (R&D output in each timestep is proportional to the rate of R&D progress in that timestep). If performing 1% more of a task would increase R&D output by x%, the task’s weight is x. Again, if R&D resources are allocated efficiently, this should correspond to weighting each cognitive task by the total wages paid to people while they perform it.

Objections talking in terms of the “% of cognitive tasks”

Objection 1: We have already automated >70% of 1700 tasks? And there was no explosion of economic growth. So why do you think this time will be different?

In economic models of automation, the growth effects of automation depend on how quickly new tasks are automated. If you automate 90% of tasks, the models I’ve seen predict GDP/capita should rise by >10X. But if this is spread out over 300 years, it might correspond to 1% growth per year. So the effect would not be extremely high rates of growth. But if this was spread out over 10 years, it would correspond to >20% growth. So the first key difference here is that I’m considering much faster rates of automation than we’ve seen historically.

The second difference is that I think we could ultimately see full automation. Many growth models predict a qualitatively different long-run outcome from this compared to mere partial automation. With full automation, models tend to predict accelerating economic growth.

Objection 2: Won’t we introduce new types of tasks into the economy? If so, then AI that can perform 100% of 2020 cognitive tasks won’t necessarily be able to perform the new cognitive tasks we’ve introduced.

I think this objection has some force.

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204 You get 10X from having all human workers concentrated on the remaining 10% of tasks, and having enough machines to increase per-task output on the automated tasks by 10X. This increases output of all tasks by 10X, so increases total output by 10X. You can get further gains from increasing the output of automated tasks even further. This all assumes you’re investing enough in capital accumulation to get enough machines to do this.

205 300 years of 1% annual growth is a 20X increase.

206 10 years of 20% growth is a 7X increase.

207 I discuss this point further in a previous report on whether AI could drive >30% GDP growth.
One attempt to dodge is to reiterate that the FTM implies that, over time, the automated tasks will become less important and non-automated tasks will become more important. We can claim that the model represents the introduction of new tasks (that AI can’t perform) via the increased importance of non-automated tasks. More precisely, the apparently new task is really just a new application of a pre-existing non-automated task and this new application makes the non-automated task more important (as the FTM predicts).

I like this dodge. I think it suggests that the FTM’s predictions need not go badly wrong because of this objection. If we think many important new tasks will be introduced that AI can’t perform, we can increase the degree to which automated tasks become less important (by decreasing the parameter $p$) and keep in mind that our training requirements for AGI (AI that can perform $\rho$ all cognitive tasks) should include any newly introduced tasks.

But I do worry that the FTM’s misleading ontology, in which the tasks needed for GDP and for R&D are fixed over time, may introduce other issues. This objection makes me view the abstraction of “% of cognitive tasks that AI can perform” less useful and meaningful.

**Objection 3:** AI will add value by enabling entirely new workflows as well as by automating existing ones.

As with the last objection, I think it has some force but there’s a dodge that I like.

The growth model allows that, in addition to replacing humans on automated tasks, AI can have *additional* economic impact by producing more output on automated tasks than humans previously produced. If AI has economic impact by enabling new workflows, we can say the model represents this via AI producing additional output and already-automated tasks. The apparently new task is just a new application of a pre-existing automated task. If we think this effect will be significant, we should increase the parameter $p$.

**Objection 4:** In practice it won’t be possible to actually measure what % of cognitive tasks AI could perform.

I agree that it will be very hard to evaluate precise claims about the “% of cognitive tasks” that AI could readily perform in each year. But it isn’t meaningless; it’s the kind of thing that economists have tried to measure. In principle you measure it by going through the concrete cognitive tasks that each person is in fact performing in each year (and how much they’re being paid to do it, as implied by the time they spend and their salary), and ask the technological question of whether AI could perform that task instead (with a limited amount of engineering work and rearranging of workflows). Any measurement would necessarily involve many arbitrary judgment calls about what to include, but that doesn’t render it meaningless or prevent us reaching rough conclusions.
Even if it didn’t map to reality well at all, the “% cognitive tasks” abstraction would still be the best way I’m aware of to model AI continuously improving from today when it (seemingly) can readily perform <1% of economic tasks to a future world where it can perform ~all cognitive tasks. A skeptic can just think of the framework as giving some arbitrary one-dimensional scale on which “AI capabilities” improve between today and AGI.

**Objection 5: Whether AI can or can’t perform a task depends in part on the extent to which nearby tasks are already automated.** But in your model, whether AI can perform a task depends solely on SOTA AI capabilities (measured via the biggest training run to date).

I agree with this. Automating one task can “unlock” a nearby task for automation by standardising the workflow. If nearby tasks have been automated, this reduces the AI capabilities needed to automate any given task.

My current interpretation of the Full Takeoff Model is that AI can “readily perform” a task if it can perform it with <1 year of engineering effort and work changing workflows, and it’s profitable to make these workflow changes. This ignores the question of “But are adjacent tasks already automated?”

I’m not sure there’s a way to get around this without significantly complicating the model. A better version might say that, if there’s been a few years since AI could perform 20% of tasks then this reduces the training requirements for AI to automate 30% of tasks, since AI automating 20% of tasks will make it easier to automate further tasks. More generally, the training requirement for performing x% of tasks fall over time once AI can perform x - e% of tasks because we expect nearby tasks to be automated.

I think the effect of this change would be to make very fast takeoff somewhat less likely (by raising the training requirements for AI suddenly performing 100% of tasks without any nearby tasks being automated) and make very slow takeoff less likely (by lowering the training requirements for eventually automating 100% of tasks via iteratively automating more and more nearby tasks).

The truth is, though, that the evidence about training requirements for different levels of automation is already extremely rough. It consists of first estimating the requirements for 100% automation (perhaps via Bio Anchors), then adjusting this based on evidence about the effective FLOP gap for lower levels of automation. My uncertainties here are already so big that this doesn’t feel like a significant contributor.
How this report relates to previous thinking about takeoff speeds

Paul Christiano’s 2018 blog post

Paul Christiano argued for a slow takeoff in an influential 2018 blog post. Most of the post counters various arguments that have been made for fast takeoff. His central argument for slow takeoff is:

- Before we have an incredibly intelligent AI, we will probably have a slightly worse AI.
- A slightly-worse-than-incredibly-intelligent AI would radically transform the world, leading to growth (almost) as fast and military capabilities (almost) as great as an incredibly intelligent AI.

I agree with this argument, but I think its conclusion is that takeoff will be continuous rather than that takeoff should be slow.

The argument precludes a discontinuous jump in capabilities or impact, because some slightly-worse AI would have caused an intermediate level of impact first. This I agree with (though I assign some probability to discontinuous jumps in capability nonetheless).

But the argument doesn’t preclude AI capabilities and impacts increasing continuously but extremely rapidly. It doesn’t speak to whether the slightly-worse AI will occur 1 year vs 1 second before the slightly-better AI. And this can make a big difference to takeoff speed. If AI capabilities improve continuously but go from today’s capabilities to AGI in one month, then it seems possible that we go straight from world GDP doubling in 24 years to it doubling in 1 year, which is a very fast takeoff by Paul’s definition. And the GDP trajectory underlying this could also be entirely continuous.

The framework of this report is (an example of) one in which AI progress is assumed to be continuous but this can still give rise to fast takeoff if the rate of continuous AI improvement is sufficiently fast. More concretely, AI capabilities improve continuously as you cross the effective FLOP gap, but if you cross that gap sufficiently quickly then takeoff is fast.

Paul predicts that takeoff will be slow enough that there’s a full 4-year doubling of world GDP before the start of a 1-year doubling (and a full 8-year doubling before a 2-year doubling, etc). The Monte Carlo analyses calculate the probability that this is the case:

<table>
<thead>
<tr>
<th>Median AGI training requirements in simulation (FLOP using 2020 algorithms)</th>
<th>Probability of a full 4-year doubling of world GDP finishing before a 1-year doubling begins</th>
<th>Probability of a full 8-year doubling of world GDP finishing before a 2-year doubling begins</th>
</tr>
</thead>
<tbody>
<tr>
<td>~1e31</td>
<td>49%</td>
<td>32%</td>
</tr>
<tr>
<td>~1e36</td>
<td>74%</td>
<td>47%</td>
</tr>
</tbody>
</table>
In a nutshell, then, my reply is: *Yes takeoff will probably be mostly continuous, but it could still be fast.*

Eliezer Yudkowsky’s Intelligence Explosion Microeconomics

*These are my opinions, and Eliezer might disagree with my characterisation of his thinking.*

Intelligence explosion microeconomics doesn’t argue for takeoff happening in weeks rather than in years.

My impression is that Eliezer Yudkowsky expects takeoff to be very fast, happening in time scales of days or months. By contrast, this framework puts the bulk of its probability on takeoff taking multiple years.

Does Eliezer give arguments for a transition taking weeks or months, rather than years? *Intelligence Explosion Microeconomics* (IEM), Eliezer’s most detailed piece on this topic, gives various qualitative arguments for thinking that an intelligence explosion would not fizzle out but instead involve intelligence growing super-exponentially. There are also arguments for thinking AI will only need to outcompete the very small fraction of humans who do AI software R&D, rather than outcompeting the whole world, for them to kickstart an intelligence explosion. But these arguments don’t (attempt to) quantify either the length of the transition to AGI or the pattern of software progress during and after the transition. So they don’t speak to whether we should expect the transition to take days vs years; and thus to whether the accelerating AI progress will take the form of a slow takeoff vs a fast takeoff. Therefore I view those arguments as all wholly compatible with the full range of scenarios sketched in this report from 1 year takeoff to 30 year takeoff.

For example, Eliezer argues that the comparison of chimps and humans suggests that returns to improving the algorithms for general intelligence are very favourable in the human range. This can be interpreted as claiming that the returns to software R&D, which I quantify with r, will be favourable around human-level AI. But the argument doesn’t (attempt to) quantify i) how good these returns are, or ii) the time it will take AI to transition from “comparably useful to software R&D as today’s AIs” to “fully automates software R&D”, or iii) the rate of software progress during and after this transition. Nor is there any attempt to make a bounding argument - to argue that any takeoff respecting these constraints must be extremely fast. Without this, the argument doesn’t support takeoff happening in days vs years.

Perhaps the closest thing in IEM is the analogy to uranium. To simplify, when the density of uranium is below a critical threshold, no chain fission reaction occurs. But once it rises even a tiny bit above that threshold, the chain reaction quickly explodes.\footnote{The threshold was $k = 1$. When $k = 1.0006$, the neutron level doubled every 2 minutes.} If AI follows an analogous trajectory then the transition from “AI can’t really help with software R&D” to “AI recursively self-improves, doubling its own abilities every few hours” would be very quick indeed. While I find the analogy suggestive of the *logical possibility* of a very quick transition, I think more work...
is needed to show that this is plausible or probable in the case of AI. (I’d be excited about someone doing this work, teasing apart why the transition is so sudden in the case of uranium and what analogous assumptions would need to hold in AI for a comparably quick transition.)

I think of this report as providing a quantitative framework for Intelligence Explosion Microeconomics

IEM qualitatively discusses a number of factors:

- Moore’s law would go faster if humans ran on computer chips.
- The importance of algorithmic improvements to AI progress.
- The (large) returns to higher quality intelligence

As I said above, he doesn’t use these factors to predict takeoff speed, or even to bound takeoff speed (e.g. he doesn’t argue takeoff must take less than 1 year to be consistent with this evidence).

This report quantities these factors, wherever possible using relevant empirical data:

- Moore’s law would go faster if humans ran on computer chips.
  - Data on the returns to hardware R&D.
    - An economic model where AI increasingly automates the cognitive work of hardware R&D.
- The importance of algorithmic improvements to AI progress.
  - Data on returns to software R&D.
    - An economic model where AI increasingly automates the cognitive work of hardware R&D.
- The (large) returns to higher quality intelligence
  - Correlations between brain size, IQ and output.
  - Data from ML on how much less thinking time models with “bigger brains” need to achieve the same performance.
- The report also incorporates the fact that AI might have strong comparative on some tasks over others, which tends to slow down takeoff speed.

So I think of the report as providing a quantitative framework to think about the factors that are discussed qualitatively in IEM.

Other arguments for discontinuous AI progress around AGI

I put ~6% on a substantial discontinuity in AI progress around the human range

I agree that takeoff could happen in mere days if there is a transition in days from “AI that can perform <20% of cognitive tasks in today’s software R&D” to “AI that fully automates software R&D” and there are no compute bottlenecks to a software-only singularity. In this case, the pace
of software progress could jump from its current pace (doubling every few years) to a 1000X faster pace (doubling in hours).\footnote{Let’s assume software currently takes \~2 years to double with \~20,000 high quality software workers. After developing AGI that’s equally good at software development as a high quality software worker, it may be possible to run \~10 million such AGIs. (This depends on the AGI runtime requirements.) This implies the first post-AGI software doubling could happen \(10 \text{ million} / 10,000 = 1000\times\) more quickly than a current software doubling. So it could happen in \(< 1\) day. If a software only singularity is possible, subsequent software doublings would be faster than this so that within a few days very many doublings have occurred: an ‘intelligence explosion’. I model out this scenario \url{here}.}

I put low probability on such a large discontinuous jump occurring, for familiar reasons. In particular

1. Such large discontinuous jumps are rare in technological progress in general, rare in software progress and rare in AI.\footnote{Though progress can be somewhat discontinuous in particular narrow applications of AI like Go, progress in entire domains (like games) is more continuous and progress in the field as a whole is even more continuous.}
2. Large discontinuities seem more plausible in narrow areas than in broad areas and achieving general intelligence seems like a very broad area (though there’s more room for doubt about how broad the skills for software R&D are).
3. It seems to me the rebuttals\footnote{See Paul Christiano’s \url{blog post} and AI Impact’s \url{page}.} of specific arguments for why there might be discontinuities around AGI are strong, and no good responses have been given 4 years on.

That said, I do think that small jumps in AI capabilities are likely to occur, and that we should attach some probability to substantial discontinuous jumps in AI capabilities.\footnote{By ”substantial discontinuous jump” I mean “\(>10\) years of progress at previous rates occurred on one occasion”.} How much probability would I assign to a large jump or kink in capabilities around AGI? Based on a rough outside view argument, I’d maybe assign \~6%?

- AI impacts looked at 38 trends, 20 of which had substantial discontinuities, \~50%
- They went looking for discontinuities though, so I’d put this probability 4X lower, \~12%.
- But the generality of AGI and the continuity of recent AI progress provides some reason to think big jumps are less likely. And even if there is a big jump, there’s no particular reason for it to happen just before AGI. So I’ll lower this another 4X: \~3%.
- I update up to \~6% based on an argument from Nate Soares about the chimp-human transition.

This has not been the main focus of my work, which was exploring the implications of a compute-centric approach that doesn’t have substantial discontinuities (beyond those implied by a small effective FLOP gap).
**Takeoff speed can differ in different domains**

Suppose it takes 1 month to go from today’s AI capabilities to a disembodied AGI, but it then takes decades for this to affect GDP due to various bottlenecks. Is takeoff fast or slow? If you measure AI capabilities by the ability to perform cognitive tasks that a human worker could perform remotely, takeoff was very fast. But if you measure AI capabilities by their impact on the economy, takeoff was slow.

Or suppose that overnight we develop AI that massively increases military power, giving its controller a decisive strategic advantage, but this AI doesn’t accelerate technological progress. In one relevant strategic sense takeoff is fast, but in another it is slow.

In cases like these, it can be useful to talk about takeoff speed in domain X. The question of takeoff speed becomes: **How long will it take to go from “AI has a minor impact on X” to “AI is making X go through the roof”.** Here are some example domains:

- **Cognitive output.** Annual output on tasks that a human worker could do remotely. E.g. software development, math, strategy, knowledge work, writing.
- **SOTA technological progress.** The speed at which we’re developing new technologies (distinct from how quickly they are integrated into the global economy).
- **Military power.** Ability to win a hot war.
- **GDP.** GDP as measured by the incomes and consumption of fleshy humans.
- **AI-inclusive GDP.** GDP as measured by the incomes and consumption of fleshy humans and digital agents (including human uploads and AIs).

We care about different domains for somewhat different reasons, and fast takeoff is more plausible in some domains than others.

**Conditions under which simple growth models predict fast takeoff in GDP**

Let’s first take a simple case where we’re not modelling technology or TFP.

In standard growth models output Y (which represents GDP) is a function of capital K and labour L. The equation is:

\[ Y = g(K, L) \]

Capital cannot replace labour in these models. There are diminishing returns to capital which makes it hard to get a large output by building more capital alone. An extreme but simple way to model this is:

\[ Y = \min(K, L) \]

These models can represent full automation from AI via capital being able to replace labour in production. After the transition to full automation there aren’t diminishing returns to capital. If you
can amass enough capital, output can become very high. To increase output significantly you’ll need enough capital to replace all the labour many times over. One simple way to model this is: \( Y = K + L \)

There are a few ways to model a continuous transition here. You can model the economy as containing many tasks, and have capital perform and continuously increasing fraction of them. (This is my approach in this report.) Or you can model the elasticity of substitution between capital and labour (as in a CES production function); it starts <1 (so that capital and labour are complements) and continuously increases until it approaches infinity (so that capital and labour are perfectly substitutable).

Fast takeoff means that the growth rate of \( Y \) increases very suddenly during the and immediately after transition (see earlier discussion). This will only happen if both:

1. The transition happens very quickly.
   - Output can only become very high once that transition has occurred and capital can replace the limited supply of labour. If the transition is gradual then the gain in \( Y \) will be spread out over time.
2. Shortly after the transition there is lots of capital, including enough to replace labour many times over.
   - Even after the transition, output won’t become significantly higher unless there is enough capital to replace labour many times over. (If one type of capital replaces labour (e.g. AI robots) and another doesn’t (e.g. trucks) then there must also be lots of both types of capital, as there are diminishing returns to each type.)

If both these conditions hold then you quickly transition from a world where capital can’t replace labour to a world where it can and there’s enough capital to replace it many times over. After the transition, the inputs to production are many times higher than before the transition.

Growth models would predict that \( Y \) increases extremely rapidly during such a transition, implying a very rapid increase in the growth of \( Y \). The simple example equations I gave clearly imply that if conditions 1 and 2 hold, there will be a very rapid increase in the growth of \( Y \).

What about technology? Well, the best models of technological progress are similar to the model of output I’ve been discussing. You simply replace \( Y \) with \( \frac{dA}{dt} \), the rate of technological progress (you also model diminishing returns, but we can set that aside). So you’ll get a sudden and dramatic increase in the rate of technological progress if conditions 1 and 2 hold for the tasks involved in technological progress. So this doesn’t really change the analysis.

How does this all relate to the broader report? The historically fast growth of both computer hardware and software give reason to think conditions 1 and 2 could hold for cognitive tasks.

- Condition 1 (fast transition) holds if we rapidly cross the effective FLOP gap, achieving 100% automation of cognitive tasks. Fast growth of hardware and software (i.e. of FLOP/$ and 2020-FLOP per FLOP) allow us to cross the effective FLOP gap quickly.
● Condition 2 (enough capital to replace humans many times over) holds if there’s enough computer capital to replace human cognitive labour many times over shortly after the transition. Fast growth of hardware and software allow us to quickly increase the numbers of AGIs we can run after crossing the effective FLOP gap.
  ○ In fact, condition 2 nearly always holds in my framework because either i) by the time we have enough FLOP to train AGI we already have enough to run 10s of billions of AGIs, or ii) around when we develop AGI the amount of 2020-FLOP is increasing extremely rapidly due to AI automation of hardware and software R&D.
  ■ For factor (i) it’s essential that compute can be easily reassigned from one purpose to another. After inventing AGI we won’t need to print new specialised AGI chips to run the new AGI algorithms.
  ○ So the main determinant of whether there’s fast takeoff of GDP is normally condition 1.

This gives an interesting perspective on why hard takeoff in GDP can happen in this framework. It’s because it allows for the possibility of a rapid transition to a world in which computer capital can replace human cognitive labour and there is very soon enough computer capital to replace human cognitive labour many times over.

If I had captured this dynamic with a different sort of growth model, I would have got a comparable result. E.g. if instead of a task-based model, I could have used a standard CES growth model with 3 inputs: human labour, computer capital, and physical capital. Rather than the ‘transition’ corresponding to AI performing a greater fraction of tasks, it would correspond to increasing substitutability between human labour and computer capital. I’d peg this substitutability to the biggest training run that has occurred. The overall result would be the same: the possibility of a fast transition to a world where AI can completely replace human cognitive labour.

This also gives an interesting perspective on a key bottleneck of the report: physical capital. While the quantity of computer capital has grown extremely rapidly in recent decades, this is unusual. The quantity of physical capital generally grows much more slowly. This means that it could prevent Y from becoming extremely large, even if there’s enough computer capital to replace human cognitive labour many times over.

Why a fast takeoff probably has its origins in the transition to AGI rather than purely in the aftermath of AGI

I’ll analyse a simple toy model of growth dynamics in the aftermath of AGI. This toy model suggests that the internal dynamics of the aftermath themselves won’t by themselves lead to a fast takeoff. But it also suggests that if the transition to AGI was sufficiently quick, there would be a fast takeoff.
Here’s the toy model. We’ve developed AGI, and can run a certain number of AGIs. Let’s say that, initially, the AGIs’ total cognitive output equals 10. For now, let’s say the only way to increase their cognitive output is via software R&D to improve the AGI algorithms (i.e. recursive self-improvement).

After a certain amount of time, the AGIs will have doubled their own cognitive output to 20. Let’s call this amount of time D₁. D₁ is the duration of the first doubling of cognitive output after AGI. (“D” for “Doubling time”.)

After some additional amount of time, the AGIs will have doubled their cognitive output a second time to 40. Let’s call this additional amount of time D₂. D₂ is the duration of the second doubling of cognitive output after AGI.

And similarly, D₃ is the duration of the third doubling of cognitive output, and D₄ the duration of the fourth. More generally, Dₙ is the duration of the nth doubling of cognitive output after AGI.

The key question for takeoff speeds is: what is the ratio between Dₙ and Dₙ₊₁? That is, what is the ratio between successive doubling times? If the ratio is very large, e.g. Dₙ = 10Dₙ₊₁, then there is a fast takeoff. We go very quickly from AI self-improving at a moderate rate to it self-improving 10X faster. But if the ratio is relatively small, e.g. Dₙ = 2Dₙ₊₁, then takeoff is slow. We move gradually from AI self-improving at a moderate rate to it self-improving at a slightly faster rate.

This metric of takeoff speeds mirrors Paul’s definition in terms of GDP doubling times, except that we’re replacing “GDP” with “cognitive output”.

Ok then, what is the ratio between D₂ and D₁? In this toy model, there’s good reason to think the ratio is <2. In particular, if there’s any diminishing returns whatsoever to increasing cognitive output, the ratio is <2.

Let’s compare D₂ and D₁. At the start of the second doubling, cognitive output is 2X higher than at the start of the first doubling. (By the definition of a “doubling of cognitive output”.) If the amount of cumulative cognitive output needed to achieve the second doubling exactly equalled that needed to achieve the first doubling, the second doubling would take half the time. With twice the cognitive output (per second), it takes half as long to do a fixed quantity of work. So if (work needed for first cognitive doubling) = (work needed for second cognitive doubling), then D₁ = 2D₂.
But in fact diminishing returns are fairly ubiquitous.\textsuperscript{213} It will probably take more absolute effort to double output the second time than it did the first time. The lowest hanging fruit, the biggest improvements that are easiest to find, will have already been taken. This implies that the second doubling will take more than half as long as the first doubling. So if (work needed for first cognitive doubling) < (work needed for second cognitive doubling), then \( D_1 < 2D_2 \). The ratio between successive doubling times is <2.\textsuperscript{214}

So, at least in this toy model, it seems like takeoff will be slow in the aftermath of AGI. The rate of recursive self-improvement will increase gradually with each doubling of cognitive output, rather than suddenly.

But this still leaves open the question of how the rate of AI improvement in the aftermath of AGI compares to the rate of improvement before AGI. If \( D_1 = 1 \) week (AI cognitive output doubles in a week immediately after AGI) but the doubling in cognitive output just before AGI took one year, then the ratio between successive doubling times on either side of AGI would equal 52! The argument above still leaves open the possibility of a very fast takeoff during the transition to, and immediate aftermath of, AGI.

This explains why I believe that a fast takeoff is unlikely to arise purely from the internal dynamics of a post-AGI world, but could easily arise from a rapid transition to AGI.

There are considerations omitted from the toy model which could lead to a fast takeoff dynamic occurring purely from the internal dynamics of a post-AGI world. If the fraction of the world’s compute used to run AIs working to improve AI algorithms increases very rapidly in the immediate aftermath of AGI, then the ratios between successive doublings could be larger. For example, if twice as much compute is used for each successive doubling, then that could double the ratio. Or, similarly, if the data AGIs can access increases very rapidly between doublings, this could also increase the ratio between successive doublings. More generally, the pattern is that there’s some input to AI development (compute, data, something else) that grows very rapidly in the immediate aftermath of AGI.

I think though, that this kind of dynamic is likely to only be significant if there is a rapid transition to AGI. If the transition lasts many decades, it seems likely that we’ll already be using AIs to improve AI algorithms and already be leveraging all the data we can to train our AIs. In this scenario, I wouldn’t expect very significant gains to be left on the table from reallocating the world’s inputs just after we cross the threshold for AGI. If so, it would raise the question: why expect those inputs to be reallocated just after AGI but not before?

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\textsuperscript{213} \textbf{Are Ideas Getting Harder to Find} finds diminishing returns in the economy as a whole and many particular areas of it. See also this blog post by Matt Clancy examining the empirical evidence. To my mind, there are also strong \textit{a priori} reasons to expect diminishing returns. If some improvements are harder to discover than others, then most simple models will exhibit diminishing returns as the easier-to-discover improvements are found earlier on.

\textsuperscript{214} This is just a bound. I make my best guess about what happens in this scenario when I assess whether a software only singularity might occur.
These arguments are not conclusive. But they do lead me to expect that, conditional on a fast takeoff, the fast takeoff probably doesn’t arise purely from the internal dynamics of a post-AGI but also relies on a fairly quick transition to AGI.

**How likely is a software-only singularity?**

Suppose you have a fixed amount of hardware that’s capable of doing a particular number of physical FLOP/s. You use this hardware to run AGIs (one or more AIs that collectively automate all cognitive labour) that do software R&D. In particular, they try to improve the algorithms that the AGIs are running on.

In this scenario, how would the total cognitive output of the AGIs change over time?

In section 5 I said this depended on i) how long it takes to double the AGIs’ cognitive output the first time, ii) how the doubling time for cognitive output changes over time.

This appendix discusses (ii), in particular whether there will be a software-only singularity with doublings becoming quicker over time and, if so, how long this might last before doublings become slower over time.

While current returns to software R&D suggest a software-only singularity would happen comfortably, returns may become worse as we approach AGI and being limited to a fixed amount of physical FLOP/s could bottleneck software progress in a couple of ways.

Overall, I’m roughly ~65% on a software-only singularity occurring, and my median best guess is that it would last for ~2-3 OOMs if it happened. What would 2.5 OOMs of a software singularity mean? My unit of software in this section is “useful cognitive output per FLOP”. So 2.5 OOMs means you can 300X the rate of progress on software development, persuasion, and any other cognitive task. One way to imagine this is that it’s as if the software improvements allowed all your AGIs to think 300X more quickly; though in fact the progress will come from a combination of “you can run more AIs in parallel” and "AIs can think in new and qualitatively better ways". (And I expect some of the progress to allow AI to do entirely new things that they previously couldn’t have done even with ~arbitrarily long to think.)

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215 In practice, I expect physical FLOP to be growing very rapidly during any period where there might be a software only singularity. However, the simplification of imagining that physical FLOP is constant is still useful. This is because it can tell us about whether software (2020-FLOP per FLOP) might grow much much faster than physical FLOP during this time. If it does so, then physical FLOP will be roughly constant on timescales over which software grows very significantly. So the question “Would there be a software singularity?” maps to the question “Would software grow much much faster than physical FLOP?”.

216 By this I mean their ability to make progress on software R&D and their output in other cognitive domains like maths, strategy, persuasion, etc. My preferred unit for cognitive output is “How many remote human workers would it take to add the same amount of value?” So if the AGIs make some software progress in one month, and you’d have needed 1000 human workers to make the same amount of progress in one month, then the AGIs’ cognitive output is “1000 remote human worker equivalents”. More.
Note, even without a software singularity I expect software progress to become extremely fast by the time we have AGI.

In the rest of this section I:

- **Recap** the mathematical condition under which a software only singularity occurs.
- **Distinguish** between pure efficiency improvements and capability improvements.
- **Argue** that a singularity via only efficiency improvements seems plausible, ~50%.
- **Argue** that including capability improvements makes it significantly more plausible, ~85%.
- **Suggest** potential bottlenecks do not rule out a software only singularity but do make it less plausible, leaving me on ~65%.

Recap: the mathematical condition for a singularity

Whether there is a software singularity depends on the returns to software R&D. These returns can be quantified by the parameter \( r \). The meaning of \( r \) is: each time cumulative software inputs double, 2020-FLOP per FLOP doubles \( r \) times. During a (potential) software only singularity these inputs are provided by AGIs and they only increase due to the AGI’s improved software.

As discussed above, the mathematical condition for a software-only singularity is \( r > 1 \). Each doubling of cumulative cognitive R&D inputs must *more* than double 2020-FLOP per FLOP.

Two types of software improvements

One type of software improvement that AGIs might make is simply to make the algorithms on which they’re running *more efficient*. The same level of intelligence is then produced with fewer physical FLOP/s. An example of this type of improvement would be *pruning*, where some of the connections in a dense neural network are removed (‘pruned’) but the performance of the system is (mostly) maintained.

This contrasts with the second type of improvement, which *increases the capabilities* of the AGIs. A greater level of intelligence is produced, perhaps with the same or more physical FLOP/s. For example, GPT-3 performs much better at a range of language modelling tasks than GPT-2.

If AGIs are trying to achieve a software only singularity, they will be able to make both kinds of improvement. They will presumably work on both improvements in (roughly) whichever combination best improves software.

I will first assess whether a software only singularity could be achieved by the 1st type of improvement alone, and then discuss the effects of the second type of change.
An efficiency only singularity

We are restricting ourselves only to *efficiency* software improvements, i.e. ones that decrease the physical FLOP/s to achieve a given capability. With this restriction, the mathematical condition for a singularity here is the same as before: each doubling of cumulative inputs must *more* than double the efficiency of AI algorithms. If this holds, then the efficiency of running AGIs (of fixed ability) will double faster and faster over time. Let’s called this an “efficiency only singularity”, which is of course an example of a software only singularity.

Estimating r from ‘AI and efficiency’

What data do we have on this? Recall that to estimate r we need data on cumulative inputs and on output.

Let’s start with outputs. *AI and efficiency*, an OpenAI blog, looks at how the runtime FLOP/s needed to achieve a given level of performance on ImageNet has changed over time. They observe an 18X decrease from 2012 - 2017. This corresponds to an efficiency doubling time of 15 months and an efficiency growth rate of 55%.

What about inputs? Tamay Besiroglu’s dissertation suggests that the number of computer vision researchers grew at 19% over the same period. If the cumulative research effort on ImageNet grew at the same pace, that implies that that $r = 2.9$. (Recall, this means that each doubling of cumulative R&D inputs doubles runtime efficiency 2.9 times.)

My impression is that similar rates of software improvement have been achieved in other domains of ML, with efficiency doublings happening every 1 - 2 years. But gathering more data points on this would be a very tractable and useful exercise.

If the value of r when we first get AGI is similar to this estimate, then there would comfortably be an efficiency-only singularity. However, there are a few reasons to think that r will be smaller than this.

1. **ImageNet inputs rose more quickly than 19%**. I don’t have data on the amount of research done specifically on ImageNet. It’s plausible that it rose faster than the number of computer vision researchers overall after 2012 did. ImageNet rose in prominence, as did approaches to it that used large amounts of compute. On the other hand, the growth of *quality-adjusted* researchers is probably slower than the growth of researchers if many new researchers entered the field. In addition, even if *annual* research inputs rose

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217 Calcs.
218 $55/19 = 2.9$.
219 For example, table 2 of OpenAI’s paper shows similar or faster software gains on other select tasks as on ImageNet (though this is for training compute, not runtime).
220 This is the relevant comparison, because any efficiency gains will allow us to run more AGIs of a fixed quality.
faster than 19%, cumulative inputs would have risen more slowly than annual inputs.\footnote{ImageNet had been going since 2010, and its predecessor since 2005 (source). Earlier work on computer vision also contributed to the stock of relevant cumulative inputs. So there would have been a notable stock of cumulative inputs in 2012. If the growth rate of annual inputs increased in 2012 then, it turns out mathematically, the growth rate of cumulative inputs is initially lower than this and catches up only after a few years.} Let’s say that quality-adjusted cumulative ImageNet inputs actually grew at a rate of 25%; this still implies \( r = 2.2 \).\footnote{55% / 25% = 2.2.}

2. **There was low hanging fruit to improve ImageNet in 2012 as algorithms were using more physical FLOP than previously.**\footnote{In particular, I believe AlexNet, the system that famously won the competition in 2012, used significantly more training and runtime compute than had historically been used.} If you suddenly have access to much more physical FLOP/s than previously, new algorithms will become available\footnote{Here’s a toy example of how this might happen. To train algorithm 1 on D data points requires physical FLOP of 10*D^1.2. To train algorithm 2 on D data points requires physical FLOP of 1000*D^0.8. The second algorithm only becomes more efficient than the first once you are using a sufficiently large number of data points. It scales better with data but has a larger up-front cost, so only becomes ‘available’ when we are using enough physical FLOP to process lots of data points.} and people won’t previously have been able to pluck the low-hanging improvements to improving them. This seems correct, but I don’t think it suggests we should use a lower value of \( r \).

   a. Firstly, the model I’m using already incorporates low-hanging fruit. Each efficiency doubling is harder to achieve than the last. (Indeed, the FTM normally predicts that the first post-AGI software doubling will take >~100X effort as a software doubling takes today, though this depends on the parameter choices.) So it is consistent with the observation that efficiency improvements were easy in 2012 and have become harder since. The objector here would have to further claim that the value of \( r \) itself, which controls the rate of diminishing returns, should decrease over time.

   b. Secondly, AGI will plausibly be in an analogous situation to ImageNet. More physical FLOP will be used to train AGI, and more physical FLOP/s to run it, than with previous systems. So you’d expect there to be low-hanging fruit here for the same reasons as with ImageNet.\footnote{In fact, this gets at an important way in which my model may underestimate the speed of software progress around AGI. It implies that the first software doubling after AGI will take much more effort than the first such doubling after ImageNet 2012, because of algorithmic progress inbetween pushing us further out the curve of diminishing returns. (~100X more effort, depending on what much software inputs increase before AGI.) But if each new OOM of physical FLOP “resets” the low-hanging fruit, then the first software doubling in each case may require equal effort. This would mean that the initial post-AGI software doublings would happen much faster than I’m predicting, and even if \( r < 1 \) there would be many very rapid doublings of software.}

3. **Minimal efforts made to make vision algorithms compute-efficient before 2012.**

   Before 2012, computer vision algorithms used much less compute. In particular, compute was a small fraction of the total costs of a project, much smaller than human labour. So there was minimal incentive to optimise algorithms for compute-efficiency. But after 2012, the cost of compute for projects rose very rapidly, increasing the financial incentives to make computer vision algorithms efficient. So there may have been one
time gains from transitioning from a “we don’t care about efficiency” to a “we do care about efficiency” regime that will not be repeated again. If we ignored these one-time gains, our estimate of \( r \) would have been lower.

I think this point has some merit, but it doesn’t seem to justify a much lower value of \( r \). There are often “one-time gains” that drive progress, and my model of software progress is really just aggregating together many such one-time gains. And, to repeat, the model incorporates diminishing returns and so it expects the one-time gains to become smaller and less common over time. And there will plausibly be comparable “one time gains” in the future: as researchers spend $1 millions and much more on training runs, the financial incentives to make AI algorithms more efficient will grow significantly. The question is whether the transition from pre-2012 to post-2012 is part of a series of one-time gains that we should include in the model as part of a pattern of diminishing returns that will continue into the future, or whether they constitute an outlier from that pattern.\(^{226}\) I currently lean towards the former. This is influenced i) by a suspicion that, even before 2012, algorithm designers in computer vision were at least somewhat concerned with efficiency, and ii) by a sense that similar rates of software progress have happened in other ML domains until the current day (2022).

If I discount 25% of the observed efficiency gains as due to a one-off effect that should be treated as an outlier, then my estimate drops from \( r = 2.2 \) to \( r = 1.7 \).\(^ {227}\)

4. \( r \) will fall as we approach ultimate limits of software efficiency, and will be lower by the time we get to AGI. There is some ultimate limit to how efficient software can become; e.g. you can’t run AGI on 10 FLOP/s. Once we reach this limit, further progress is impossible. This corresponds to \( r = 0 \).\(^ {228}\) So \( r \) falls towards 0 as we approach ultimate limits, and may have fallen somewhat by the time we get to AGI.\(^ {229}\)

The longer your AI timelines, the stronger this argument as there is more time for software to approach ultimate limits before AGI.

This seems broadly correct to me, and I expect \( r \) will be lower when we get AGI than today. I don’t expect this effect to be huge because I don’t think we’ll have reached ultimate limits by the time we get to AGI (more). In addition, AGI will probably be trained using more physical FLOP and run using more physical FLOP/s than previous systems. So it seems unlikely that the first AGIs will be maximally efficient, given our lack of

\(^{226}\) One way to settle this is empirical. Look at whether rates of software progress were significantly higher just after 2012 than in periods since; if so it suggests the transition was an exception. Another way is speaking to practitioners in the field about whether they feel there is a continued pattern of this kind, or whether there was a regime change around 2012 that will never be repeated.

\(^{227}\) \( 2.2 \times 0.75 = 1.65 \).

\(^{228}\) Each doubling of cumulative inputs causes 0 doublings in efficiency.

\(^{229}\) An important question is whether \( r \) falls based on our linear distance from the limit, or our log-distance from it. If the former, then only in the last OOM of software improvement will \( r \) fall to 0 and \( r \) probably won’t change much before AGI. If the later, then \( r \) will fall somewhat during each software doubling along the way and \( r \) may decrease significantly before AGI.
experience optimising systems with that level of compute.\textsuperscript{230} I think we’ll have made 2 - 3 OOMs progress, with more than this still remaining before reaching ultimate limits.

My guess would be that $r$ falls $\frac{1}{2}$ of the way towards 0 by the time we get AGI; so my estimate drops from $r = 1.7$ to $r = 1.1$.\textsuperscript{231} I think people could reasonably expect $r$ to fall $\frac{1}{2}$ or even $\frac{2}{3}$ of the way towards 0, which would imply $r < 1$.

After considering those 4 objections, my best guess for $r$ fell from 2.9 to ~1. This matches my gut feeling that, once we have AGI, returns will be worse than the naive ImageNet data suggest, but not way way worse, and that means there could well be an efficiency only singularity. I’m about 50 -50 on whether $r > 1$ at this point. (Later, I’ll discuss how many software doublings a singularity might last for, if it happens.)

Estimating $r$ from Computational Limits of Deep learning

\textbf{Thompson et al. (2022)} find that “3 years of algorithm improvement is equivalent to an increase in computing power of 10X” in image models. This corresponds to a growth rate of 77%. The paper does not estimate growth of inputs, but using the 19% from above implies $r = 4$. This is higher than the equivalent $r = 2.9$ we estimated previously. Applying the same penalties as in the last section would leave us on $r = 1.5$.

Estimating $r$ from ‘How Fast do Algorithms Improve’

\textbf{How Fast do Algorithms Improve}, by Sherry and Thompson, is another source of data on efficiency improvements. They survey a wide range of algorithms, most of which are not specific to machine learning, and calculate the annual rate of efficiency improvement. The rate of improvement depends on the size of the problem - how many examples or data points must be processed. They find that, at a problem size of $n = 1$ billion, the efficiency of the median algorithm had a growth rate of 25%.\textsuperscript{232}

To estimate $r$, we also need data on software investments across this period. A couple of data sources imply software investment, measured in real $\$, grew at a rate of 6 - 14% during this period. Let’s say the number of quality adjusted researchers had a growth rate of 10%.\textsuperscript{233} That implies $r = 2.5$.

\begin{itemize}
\item \textsuperscript{230} Although if you think that AGI will consist of multiple interacting AIs, we may have already trained most of those AIs before training the final AI that allows the AIs to collectively perform all tasks. (OTHO, the tasks performed by the final AI could well be the main bottleneck, so that final AI’s capacity for improvement may be most relevant.)
\item \textsuperscript{231} $1.65 \times \frac{2}{3} = 1.1$.
\item \textsuperscript{232} They say “28% per year” on p.5, which corresponds to a growth rate of 25%: $e^{0.25} = 1.28$.
\item \textsuperscript{233} This is probably slightly too high, as it looks like real $\$ grew at ~10% and so quality adjusted people probably grew ~2% more slowly (due to rising real salaries). As a result, the estimate of $r$ will be slightly too low.
\end{itemize}
This methodology avoids objections 1, 2 and 3 from above, as these were specific to the ImageNet data being used. Objection 4, that returns may be worse once we get to AGI, still applies. The same ⅔ adjustment as before leaves us at $r = 1.6$.

There are a couple of big uncertainties here.

Firstly, Sherry and Thompson observe very large disparities on progress in different types of algorithms. If we have similar uncertainty about AGI we should be open to an efficiency-only singularity happening comfortably, or to it not happening at all.

Secondly, the result is sensitive to the problem size used. A problem size of 1 million, rather than 1 billion, reduces the median rate of progress from 25% to 14%, which would leave us at $r = 0.9$. I don’t know a principled way to choose the problem size. I think using the size of current SOTA AI models (e.g. # params or # data points) would imply a somewhat higher problem size than 1 billion. The problem size will be larger still for AGI.

I see this second estimate as broadly consistent with the first; both suggest $r = \sim 1$ is plausible for efficiency improvements around AGI. I find the second estimate slightly less informative because it looks at algorithms in general rather than focussing on AI.

In both cases, my biggest uncertainty is how much $r$ will decrease between today and when we get AGI. It seems fairly clear that today $r > 1$ by some margin, but that could easily stop being the case by the time we get to AGI.

A software-only singularity (including capability improvements)

The above analysis assumed we were restricted to only using software improvements that increase the efficiency of running systems with ~fixed capabilities. I guessed there was a ~50% chance these improvements would happen increasingly quickly; and if so, that might be ~10X total improvement before progress began to slow.

In reality, AGIs would also try to make improvements to make more capable AI, perhaps running on as much or more FLOP/s. This could only increase the chance of a software-only singularity occurring and its duration.

How large might this effect be? I think there are good reasons to think the effect will be big:

- New capabilities are plausibly a much bigger source of progress than efficiency improvements on existing capabilities.

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234 They say (p.4) that just under half the families show little to no improvement, while 14% of algorithms improved by more than 11X each year (on average).

235 $e^{0.14} = 1.15$.

236 $1.6 \times \frac{14\%}{25\%} = 0.9$.

237 This gives another reason to think there won’t be extremely harsh diminishing returns at this time (larger problem sizes lend themselves to faster algorithmic improvements, measured in % terms).
OpenAI argue\textsuperscript{238} for this. For example, they suggest that AlexNet – the system that famously won ImageNet in 2012 – achieved its level of performance much more efficiently than pre-existing algorithms could have. More generally, they argue that the first time a capability is achieved, the algorithm used is typically \textit{much} more efficient than pre-existing algorithms at achieving that capability. Their arguments seem reasonable to me; the key question then becomes how useful these new capabilities are. (Who cares about dramatically increasing the efficiency of new capabilities if those capabilities aren’t useful?)

My impression is that the growth in AI’s economic importance since 2012 has mostly come from new capabilities, rather than merely from increasing the efficiency of capabilities that already existed before 2012.\textsuperscript{239}

There is very large variation among humans in terms of effectiveness at software R&D.\textsuperscript{240} This suggests that, around the human level, there are very large gains to software R&D from increased capabilities.

More speculatively, perhaps AGI whose (collective) capabilities surpass any human will identify new kinds of software improvements that humans cannot see. Perhaps many such improvements will exist, as humans haven’t been able to see any of them. If so, there could be extremely rapid progress once AIs surpass the best humans.

Overall, this gives me a prior that capability improvements will be a much bigger deal than efficiency improvements during a software-only singularity. So if we previously thought $r = 1$ for efficiency improvements only, you might think $r = 3$ when you include capability improvements (so that capability improvements are twice as big a deal as efficiency improvements).

Grace (2013) measures algorithmic progress in 6 domains, and finds that in many areas about half of all progress is due to software and half due to hardware.

I believe that these improved algorithms often used constant or increasing amounts of compute, so her evidence speaks to non-efficiency gains from software.

One extremely hacky way is to assume that her measure of ‘hardware progress’ maps to increases in FLOP/$, and assume that inputs to software R&D have grown at the same rate as inputs to hardware R&D. Then the returns to software R&D will be the same as we calculated earlier for hardware: $r \approx 7$.

This is very far above the threshold for a singularity ($r = 1$). In the last section I adjusted the efficiency estimate down from $r = 2.9$ to $r = 1.1$ based on a few

\textsuperscript{238} Section 5.3 of the paper that accompanied the AI and Efficiency blog.

\textsuperscript{239} People could dig into this: the specific use cases that generate revenue and the algs used for them.

\textsuperscript{240} I’ve heard there is SMPY data on this but I couldn’t find them with an hour or so looking. Salary differentials are indicative, but they may underestimate true productivity differences for social reasons.
objections; similar adjustments here\textsuperscript{241} would still leave the overall estimate at $r = -2.5$.\textsuperscript{242}

- An important caveat, discussed more below, is that these returns might not be possible without increasing the amount of physical training FLOP and runtime FLOP/s.

- Eliezer Yudkowsky argues that the evolution of humans suggests that there are favourable returns to improving the algorithms for general intelligence around the human range, and that those returns aren’t sharply diminishing. Roughly speaking, this corresponds to the claim that $r$ is large when software reaches human levels. It’s hard to translate this into a quantitative claim, but the next footnote argues from Yudkowsky’s claim to the conclusion that $r > 2$ (around the human range).\textsuperscript{243}

- I find this evidence fairly unconvincing for the same reasons given in Paul Christiano’s blog post. Selection for intelligence, in particular for learning from and communicating to others, may have increased significantly during this period due to the massively increased importance of culturally accumulated knowledge for survival.\textsuperscript{244}

- Evidence from within ML suggests ‘cleverer’ models make much better use of compute.

  - Jones (2021) finds that, when training AlphaZero on the game Hex, using 10X more training compute reduces the runtime compute\textsuperscript{245} needed to achieve a given test result by 15X. In other words, a model that is “10 times smarter” (as quantified by its training FLOP) can achieve the same result with 15X less thinking (as quantified by its runtime FLOP).\textsuperscript{247}

\textsuperscript{241} Are these objections applicable? The first (inputs rose more quickly than we assumed) might apply if the problems studied received faster growing investment than is typical for software. The second (low hanging fruit due to more physical FLOP) applies more strongly as I believe the amount of physical FLOP by the systems studied in Grace (2013) was continually increasing. (By contrast the physical FLOP used on Imagenet didn’t increase after 2012 in the OpenAI data.) The third objection (minimal effort to make algorithms compute-efficient before 2012) doesn’t apply, as it was specific to 2012.

\textsuperscript{242} $7 \times 1.1 / 2.9 = 2.65$

\textsuperscript{243} Yudkowsky argues that the effort needed to increase intelligence didn’t significantly increase during the evolution from Australopithecus to Homo erectus to Homo sapiens. In this period, he claims, brain size increased by a factor of four. If software increased by a similar factor over this period (i.e. if better software was responsible for the same share of cognitive improvement as bigger brains), then software too increased by 4X. According to the model of this report, the effort needed to improve software increased by $4^{(1/r)}$ during the period. Suppose we accept Yudkowsky’s claim that the effort needed didn’t increase much; let’s commit to saying it increased by <2X. To meet this commitment, we’d need $4^{(1/r)} < 2$, which implies $r > 2$.

\textsuperscript{244} The Secret of Our Success is the best account of the importance of culture to the biological evolution of humans that I’m aware of.

\textsuperscript{245} The reduction in runtime compute comes from reducing the depth of the tree search. (It must be reduced by more than enough to compensate for the model being larger.)

\textsuperscript{246} This implies that, with a fixed budget for both training and runtime, it’s optimal to spend ~55% on training and ~45% on runtime. (The system is Cobb Douglas: test result = Train$^{0.55} \times$ Runtime$^{0.45}$. I have verified that this Cobb Douglas equation roughly reproduces Jones’ results.)

\textsuperscript{247} We can relate this to model size, i.e. FLOP/s at runtime, if we make an assumption relating training FLOP to runtime FLOP/s. Let’s assume that when you double model size you need 4X the training FLOP. In this case, 10X more training FLOP corresponds to a ~3X bigger model. Jones’ result is then that a 3X bigger model achieves the same result with 15X less runtime FLOP/s. This means it thinks for 45X
○ On a simple intuitive level, this suggests that the returns to training more capable AIs could be large. Increases in training FLOP, or simply training efficiency, could result in significantly more capable AIs.

○ A simple toy model supports this intuition.
  ■ To simplify the model, let’s suppose 10X more training FLOP reduces the runtime FLOP needed for a given result by 10X, rather than 15X. I.e. doubling training FLOP means that only half the runtime FLOP is needed to achieve a given result.
  ■ Suppose we have 1000 FLOP available to us in each timestep. Each timestep we must use the latest algorithms to train AGI from scratch and then run AGIs to improve AI algorithms.
  ■ It turns out that, given our assumptions, it’s optimal to use 50% of our FLOP on training and 50% on runtime. 500 FLOP each.
  ■ First, let’s walk through an example where we don’t increase training efficiency and so don’t train more capable AIs.
    ● Suppose that in the first timestep AGIs double cumulative R&D inputs. Further, assume that this doubles runtime efficiency (i.e. assume $r_{\text{runtime}}=1$) but doesn’t change training efficiency (assume $r_{\text{training}}=0$). The new algos take the same amount of FLOP to train, but run on half as many FLOP/s.
    ● Then in the second timestep we’ll again use 500 FLOP to train AGIs of the same ability, but we can run twice as many of them with the other 500 FLOP. We get twice as much R&D done as in the first timestep, so we double cumulative R&D inputs again. As before, this doubles runtime efficiency ($r_{\text{runtime}}=1$) but doesn’t change training efficiency ($r_{\text{training}}=0$).
    ● The process continues: in the third timestep we again use 500 FLOP to train our newly designed AGIs, use the other 500 FLOP to run 4X as many AGIs as in the first timestep, double cumulative R&D inputs again, so double runtime efficiency ($r_{\text{runtime}}=1$) but don’t change training efficiency ($r_{\text{training}}=0$).
    ● The process can continue indefinitely. We chose the knife-edge $r=1$, and so the software doubling times are constant over time. If we’d chosen $r > 1$, each doubling would have taken less long.
  ■ Now let’s walk through an example where we do increase training efficiency.
    ● Like last time, AGIs double cumulative R&D inputs on the first timestep and this doubles runtime efficiency ($r_{\text{runtime}}=1$). This time let’s assume it also doubles training efficiency ($r_{\text{training}}=1$). The new algos take half as much FLOP to train and run on half as many FLOP/s.

less long! Or, equivalently, a 2X bigger model achieves the same result with 6X less FLOP/s and 12X less thinking time.
Then in the second timestep we could train AGIs of the same capability with only 250 FLOP, and use the other 750 FLOP to run them twice as efficiently as before. We would get three times as much R&D done as in the first timestep. (2X as efficient, using 1.5X the runtime FLOP.) Call this option 1.

Alternatively, we could use 500 FLOP for training. Compared to the option 1, this doubles training FLOP. We will train more capable AIs than in option 1. How much more capable? Above, we assumed that doubling the training FLOP halves the runtime FLOP needed to achieve a given result. (This was based on Jones (2021).) So our more-capable AGIs will achieve the same output with half as many runtime FLOP, compared to option 1. We could achieve the same software progress as option 1 by running them with 375 FLOP, but in fact we can run them with 500 FLOP. This means we’ll get 500/375 = 4/3 times as much R&D done as in option 1, and four times as much as in the first timestep. Call this option 2.

There’s a factor of 2 from reducing the runtime FLOP of our old AGIs, and a factor of 2 from using more efficient training to train more capable AGIs. Based on the result from Jones (2021), improvements to runtime and training combine multiplicatively.

Option 2 is better than option 1, by a factor of 4/3, because it exploits the ability to train smarter AGIs. The actual numbers from Jones (2021) suggests the true effect of increasing training efficiency would be slightly larger. (He found 10X more training compute drives 15X more runtime efficiency, whereas we assumed it would drive only 10X.)

In this toy model, there’s a software only singularity just if \( r_{\text{runtime\_efficiency}} + r_{\text{training\_efficiency}} > 1 \); we saw earlier that this happens via training a bigger model (one with more FLOP/s) than in option 1. Remember, option 1 itself involved AGIs using half as many FLOP/s as in timestep 1, so in option 2 AGIs will use more than half as many FLOP/s. How much exactly? If we assume model size goes with \( \sqrt{\text{training FLOP}} \) then the model size will be \( 0.5 \times \sqrt{2} = \approx 0.7 \) times as big as in timestep 1. So models still get smaller, but they also get smarter, due to the combined effects of training improvements and runtime improvements.

By analogy with Jones (2021), this happens via training a bigger model (one with more FLOP/s) than in option 1. Remember, option 1 itself involved AGIs using half as many FLOP/s as in timestep 1, so in option 2 AGIs will use more than half as many FLOP/s. How much exactly? If we assume model size goes with \( \sqrt{\text{training FLOP}} \) then the model size will be \( 0.5 \times \sqrt{2} = \approx 0.7 \) times as big as in timestep 1. So models still get smaller, but they also get smarter, due to the combined effects of training improvements and runtime improvements.

[Weedsy fn.] We are applying the result from Jones (2021) in a subtly different context here. The original result showed that doubling the physical training FLOP (slightly less than) halved runtime FLOP to achieve a given result. There was only one algorithm used (AlphaZero). Here we are again imagining doubling the physical training FLOP, but we also imagining that we just halved training FLOP by making algorithmic improvements. You could object that the doubling training FLOP won’t halve runtime FLOP if you’ve just made some algorithmic improvement to make training more efficient. Maybe that efficiency improvement only improves training at the new smaller scale, but not so much at the original scale? This objection doesn’t seem convincing to me. My guess is that the training algorithms developed since AlexNet (the 2012 ImageNet system) also function well at the training FLOP used for AlexNet. Much more significant to my mind is the fact that Jones (2021) is a toy environment, while we’re here imagining AI that can do 100% of cognitive tasks.
that an efficiency only singularity occurs just if \( r_{\text{runtime\_efficiency}} > 1 \). So this toy model suggests that a software only singularity is considerably more plausible.

- Above I estimated \( r_{\text{runtime\_efficiency}} = 2.9 \), based on data about ImageNet (though revised it downwards to 1.1). What about \( r_{\text{training\_efficiency}} \)? The same AI system driving the runtime estimate produces an estimate of \( r_{\text{training\_efficiency}} = 3.2 \),\(^{250}\) which a similar discount would reduce to 1.2.

- More generally, most improvements in runtime efficiency also increase training efficiency,\(^{251}\) but not vice versa.\(^{252}\) So I’d expect \( r_{\text{training\_efficiency}} > r_{\text{runtime\_efficiency}} \). This implies that we get a software singularity as long as \( r_{\text{runtime\_efficiency}} > 0.5 \). This is definitely the case now, and I expect it will still be true when we get to AGI, but I’d still assign >25% to the contrary.

- All this is to say that a toy model implies that being able to train cleverer models would make a software-only singularity significantly more plausible. It uses a tradeoff between runtime and training FLOP that Jones (2021) observed in a toy environment, but that type of tradeoff does seem plausible.

- This suggests that if we thought \( r = 1 \) only including runtime efficiency improvements, we should think \( r > 2 \) once we include training efficiency improvements that can lead to more capable models. (Because in the toy model the contribution of the latter was expected to be bigger, \( r_{\text{training\_efficiency}} > r_{\text{runtime\_efficiency}} \).)

- If we discover a learning algorithm that scales as efficiently with training and runtime FLOP as the human lifetime-learning algorithm, then it seems plausible we could do a software-only singularity just by making that algorithm more efficient.

- Correlations between brain size and IQ, and IQ and productivity, suggest a relationship between brain size and productivity in humans. In particular, a 10% bigger brain is ~5 IQ points smarter, and so ~30% more productive. Extrapolating heroically, a 2X bigger brain is ~8X more productive.

- Suppose you had (an inefficient version of) the human learning algorithm, and were able to make it 2X as efficient. That would mean that, using the same amount of physical FLOP as before, you could train and run a model that was like a “2X bigger brain” and so was 8X more productive.

- Whether you succeed in doing a software-only singularity or not depends on whether you become faster or slower at making 2X efficiency improvements of

\(^{250}\) While the system’s runtime efficiency increased 18X (growth rate 55%), its training efficiency increased by 21X (growth rate 61%). With the same assumption that cumulative inputs grew at 19%, this implies \( r_{\text{training\_efficiency}} = 61/19 = 3.2 \).

\(^{251}\) Training consists in doing multiple forward passes. If you increase runtime efficiency, you decrease the compute for each forward pass.

\(^{252}\) For example, improving the optimiser or the hyper parameters don’t affect runtime efficiency.
that kind. To succeed, each 2X efficiency improvement must take <=8X as much effort as the last.

- This condition will hold unless diminishing returns to efficiency improvements are much steeper than they are today.
  - For context the estimate of \( r_{\text{efficiency}} = 2.9 \) from Imagenet models corresponds to each 2X efficiency improvement taking 27% more effort than the last. Much less than 8X more effort!\(^{253}\)
  - We can use a model like before, where \( r_{\text{brain}_\text{alg}} \) means: when you double cumulative R&D inputs you double the efficiency of the human learning algorithm \( r_{\text{brain}_\text{alg}} \) times. The condition for software-only singularity is \( r_{\text{brain}_\text{alg}} > 0.3 \). This is a lot lower than the estimates we’ve been seeing.
    - Translating back to the condition on overall \( r \) (‘When we double cumulative software R&D inputs, how many times do we double productivity?’), I see this as evidence that \( r > 1 \), perhaps comfortably so.
    - Here’s a toy model of this dynamic.
  - A qualification here is that perhaps the human learning algorithm scales well within the human range of variation (±10%), but no further. Or perhaps by the time we find anything that scales this well, we’ll have already hit the ultimate limits to software. On the other hand, you might think we could get better scaling than the human learning algorithm by scaling data in proportion to model size. (The human learning algorithm keeps data fixed as brain size increases.)

I said I was 50-50 on an efficiency only singularity happening, at least temporarily. Based on these additional considerations I’m now at more like ~85% on a software only singularity. And I’d guess that initially \( r = \sim 3 \) (though I still think values as low as 0.5 or as high as 6 as plausible). There seem to be many strong ~independent reasons to think capability improvements would be a really huge deal compared to pure efficiency problems, and this is borne out by toy models of the dynamic.

How long might a software-only singularity last?

Even if a software only singularity occurs, there’s a further question of how much software improves before software doublings start to slow down. I don’t have much to say here. There are a few sources of evidence that I’m aware of:

- **How big were the total efficiency improvements on ImageNet?** Runtime efficiency increased 18X from 2012 to 2017; training efficiency increased 44X from 2012 to 2019. Perhaps returns to increasing the efficiency at which we achieve AlexNet-level performance become much worse shortly after this (though returns for making more capable models more efficient might be better). We can anchor to this and predict total gains of \( 2 - 5 \text{ OOMs} \) before returns become worse and doublings start to slow down.

\(^{253}\) \( 2^{(1/2.9)} = 27\% \).
○ Why 2 OOMs? Conservatively only include training efficiency increases (44X) and assuming these ran out soon after 2019 (at 100X).
○ Why 5 OOMs? Combine training and runtime increases multiplicatively as in the \textit{toy model} above: $18 \times 44 = 800$, ~3 OOMs. Then assume the trend could continue for another ~2 OOMs before running out.

- \textbf{How far away are ultimate limits to software efficiency (at this level of physical FLOP)?}
  ○ \textit{Runtime efficiency}.
    - I expect that when we first train AGI, its runtime efficiency will be less than the human brain. The first version of an AI system with a new capability is typically not well optimised for runtime efficiency. AGI might initially be 10X or 100X less efficient than the human brain, perhaps much more.
    - In addition, I’d guess that the ultimate limits for runtime software efficiency are significantly better than that of the brain:
      - The brain does specialised cognitive tasks using general thinking software that is much less efficient than specialised software would be (e.g. doing mathematics using neural networks).
      - There’s significant variation between humans in IQ, even holding brain size fixed.
      - In evolutionary time, we have not had brains our size for that long; and they have not been optimised for doing the cognitive tasks needed for science for long.
      - AI will have a some significant structural advantages over humans that make them more productive; e.g. faster serial speed, no leisure (though there are potential ethical concerns here), more motivated to work hard and coordinate effectively. \textit{More}.
  - Overall, \textbf{3 OOMs} or more increase here seems likely before hitting limits.
  ○ \textit{Training efficiency}.
    - When we first train AGI, its training efficiency will be \textit{many} OOMs below human learning efficiency. Human lifetime-learning takes ~$1e24$ FLOP\textsuperscript{254}, and training AGI with $1e30$ FLOP would be less than my median. Naively, that suggests 6 OOMs improvement available just in training efficiency.
      - Even if the human learning algorithm is extremely complicated and evolution has learned thousands of clever tricks, in principle AI could discover and hardcode them themselves.
    - It seems like human learning efficiency is not close to physical limits.
      - We could do much better to fully optimise people’s experiences for learning, e.g. by providing better and more personalised learning curricula.
      - Again, there’s large variation in learning efficiency between humans.

\textsuperscript{254} Quoting from \textit{Bio Anchors}: “I took the anchor distribution to be the number of total FLOP that a human brain performs in its first 1 billion seconds (i.e. up to age ~32); my median estimate is (1e15 FLOP/s) * (1e9 seconds) = 1e24 FLOP.”
Again, there’s only a limited amount of time (on evolutionary timescales) that human-sized brains have been optimised for learning. And much less time still being optimised to learn in our current cultural environment (e.g. from books).

So if there’s 2 - 5 OOMs of software gains still to be had once we get AGI, perhaps returns become worse (ending the software singularity) after ~2 OOMs.

- Overall, 5 OOMs or more increase here seems likely before hitting limits.
  - Bigger brains
    - Even if human brain learning and runtime efficiency is at physical limits, you could increase total productivity simply by training bigger brains. Above I discussed the naive estimate that doubling brain size would 8X productivity; this means 4X more output per FLOP.
    - If we trained 100X bigger-than-human brains using the human lifetime learning algorithm, this would take 100X the compute (people with bigger brains don’t take longer to learn), 1e26 FLOP. That would increase productivity by 10,000X, 4 OOMs.
  - What do ultimate physical limits tell us about how long the software-only singularity will last?
    - If initially r = 2 and we’re Y OOMs from ultimate physical limits to software, and the software-only singularity requires r>1, then a really simple model might say that the singularity will last for Y/2 OOMs. The idea is that r=0 at ultimate limits, and we assume it falls a constant amount towards 0 with each OOM of software improvement.
    - We guesstimated 3 OOMs to improve runtime software efficiency, 5 OOMs to improve training efficiency and at least 4 OOMs to increase output by training bigger models. I think these are multiplicative, training and runtime improvements are multiplicative in the toy model above, and then training bigger brains is clearly a distinct type of improvement.

- Directly estimate r at human levels of software. Above I discussed Yudkowsky’s claim that returns to software aren’t sharply diminishing around the human level of intelligence, and suggested we could very roughly parse the claim as r > 2. This implies
that, even when software is at human levels, returns would have to become notably worse before a software only singularity stopped.

I don’t trust any of these lines of evidence, but my best guess is that, based on the evidence discussed so far, a software singularity, if it started, would last ~5 OOMs before software doublings become slower. It could be 1 OOMs, or even 10 OOMs.

Importantly, even after software doublings become slower \( (r < 1) \), there may still be very fast software progress for multiple doublings. For example, if \( r = 0.5 \) then each doubling takes twice as long as the previous one. If the fastest software doubling took 1 week, after which each doubling is twice as long as the previous one, there would still be 16X software progress over the next 30 weeks.\(^{256}\)

**Bottlenecks from a fixed supply of physical FLOP**

Two bottlenecks are salient to me.

1. The need to experiment to find better algorithms
2. The dependence of software progress on using more physical compute.

**The need to experiment to find better algorithms**

As discussed in section 6, one seemingly important contributor to software R&D is doing experiments to see which algorithms have good performance in practice. Across all the data series considered in this section, the physical FLOP available for doing such experiments was increasing exponentially while software progress happened. Perhaps this exponential growth in physical FLOP was needed to run enough experiments to maintain the observed pace of software progress. Perhaps we’d have seen slower software progress if the amount of physical FLOP had remained constant (as it would in a software singularity). If so we’d have estimated a lower value for \( r \) and judged the software singularity to be less likely.

For example, I estimated that \( r = 2.9 \) for runtime efficiency improvements on Imagenet. But perhaps we’d have only seen half these improvements had the physical FLOP used for experimentation remained constant. In which case I’d have instead estimated \( r = 1.45 \), and lowered my probability of a singularity accordingly. And similarly, perhaps the software improvements observed in Sherry and Thompson (2021) and Grace (2013) would have been smaller had the physical FLOP for experimentation remained constant. Again this would reduce the estimates of \( r \) that I derived and make a software singularity look less likely.

To make the potential bottleneck here concrete, let’s imagine trying to achieve an efficiency-only singularity. Each doubling of efficiency will require a certain number of experiments. We can compare the number required for one efficiency doubling with the number required for the next efficiency doubling. The key question is: How does the number of experiments required change

\(^{256}\) The next four doublings take 2 weeks, 4 weeks, 8 weeks, and 16 weeks. That’s \( 2^4 = 16 \)X progress in 30 weeks.
for successive efficiency doublings? If we needed a constant number of experiments to achieve each efficiency doubling, the physical FLOP needed for experimentation would actually decrease over time. After the first doubling, each experiment would take half as much physical FLOP.\textsuperscript{257} If we needed twice as many experiments for each new efficiency doubling, the physical FLOP needed for experimentation would be constant over time. Each successive doubling would require twice as many experiments, but each experiment would use half as much compute. The effects would cancel. Lastly, if we needed more than twice as many experiments for each new efficiency doubling, the physical FLOP needed for experimentation would increase over time.

If we instead imagined a software-only singularity that included improvements in the capability of AGIs, then this analysis would shift. In the previous paragraph, after each software (efficiency) doubling, the physical FLOP per experiment halved. But capability improvements would make experiments more computationally expensive. So the physical FLOP per experiment would not halve after each software doubling; it might decrease more slowly than this, or even increase if new models use more physical FLOP/s.\textsuperscript{258} This makes it comparatively more likely that experiments would significantly bottleneck progress.

Still, we could get significant capability gains while doing a fixed number of experiments per software doubling, by holding physical runtime FLOP/s fixed. And we can adjust how we conduct software R&D to reduce the reliance on large experiments (e.g. conducting experiments on a smaller scale, reasoning more from first principles, inferring the outcome of a training run from the first 100 timesteps, a move back to “good old fashioned AI” where AI runtime software is handwritten). I think experiments would probably eventually bottleneck capability improvements, but this might not happen until we’ve seen multiple OOMs of improvements.

One way to model this would be to have physical FLOP perform some fraction of software R&D tasks;\textsuperscript{259} this input would stay fixed during the software singularity and so eventually (if R&D tasks are complementary) bottleneck progress. I believe\textsuperscript{Epoch} are investigating a model of this kind.

My takeaways from the previous few paragraphs are:

- The number of experiments for each software doubling has to increase at a fast exponential rate for it to block an efficiency only singularity. This doesn’t seem very likely.
- It is more likely to be a bottleneck for capability increases, but this is not guaranteed.

\textsuperscript{257} I’m assuming that the physical FLOP required for an experiment is proportional to the runtime FLOP/s of the system the experiment is investigating.

\textsuperscript{258} This depends on the balance between runtime efficiency improvements and capability improvements, and on how capability improvements affect the AGI’s runtime FLOP/s. If we are increasing the physical FLOP/s of our SOTA AIs, then we will have fewer experiments at that scale; but capability improvements can also come from using a fixed amount of FLOP/s more effectively.

\textsuperscript{259} You could use the fraction of lab spending on physical FLOP vs talent to decide the fraction of software R&D tasks performed by physical FLOP. (Although physical FLOP allows labs to develop and run AIs, as well as improving their algorithms; so this is problematic.)
Overall, I’d be surprised if experimental bottlenecks block a software-only singularity in its early stages, but wouldn’t be surprised if they blocked it after a couple of OOMs of improvements.

I think this consideration should lower our estimates of \( r \); if I had to say it would lower \( r \) from 3 to 2.

It also lowers my probability that a software-only singularity will occur at all from \( \sim 85\% \) to \( \sim 70\% \) and makes me think any software singularity would last less long (\( \sim 2-3 \) OOMs rather than \( \sim 5 \) OOMs).

The dependence of software progress on using more physical compute

A decent chunk of software progress may be the result of software adapting to larger hardware scales (h/t Paul Christiano). In other words, there are fast diminishing returns to improving algorithms that use a fixed budget of (physical) FLOP/s, but using more FLOP/s allows us to find new algorithms that are much better adapted to the additional FLOP/s than our previous algorithms.

As a concrete example, suppose \( \text{alg1} \) has efficiency 100 when run on 1e9 FLOP/s. \( \text{alg2} \) has a very similar efficiency of 105 when run on 1e9 FLOP/s. But when run on 1e10 FLOP/s, \( \text{alg2} \) has an efficiency of 200, compared to \( \text{alg1} \)’s efficiency of 110.

<table>
<thead>
<tr>
<th>Efficiency of algorithms</th>
<th>( \text{alg1} )</th>
<th>( \text{alg2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1e9 FLOP/s</td>
<td>100</td>
<td>105</td>
</tr>
<tr>
<td>1e10 FLOP/s</td>
<td>200</td>
<td>110</td>
</tr>
</tbody>
</table>

\( \text{alg2} \) is much better adapted to the new FLOP/s budget than \( \text{alg2} \), even though their performance was similar on the old budget.

If much historical algorithmic progress is of this sort, then algorithmic progress would become much slower if our budget of FLOP/s remained constant (as during a software-only singularity).

There’s a couple of reasons it could be easier to improve algorithms are larger hardware scales:
- Less effort has been made to optimise algorithms for that large scale historically, so there’s more low-hanging fruit.
- Improvements in scaling behaviour (e.g. moving from \( O(n^2) \) to \( O(n \log n) \), or moving to Chinchilla scaling) have bigger effects at larger levels of FLOP/s.

How does this consideration affect the estimates of \( r \) that I’ve used in this section?

---

\(^{260}\) *How Fast do Algorithms Improve* supports the idea that we’ve only maintained our overall pace of algorithmic progress by increasing our physical FLOP budgets. It finds that algorithmic progress is faster at larger problem sizes.
• The *AI and Efficiency* and the *How Fast do Algorithms Improve* estimates are affected in similar ways.
  ○ They both measure algorithmic progress as a *reduction* in the FLOP/s to achieve a given capability. Their measured software progress does *not* rely on using new algorithms that are better adapted to new scales of FLOP/s.
  ○ *However*, the fast software progress they measure may be a result of adapting to a new large hardware scale, as happened with ImageNet in 2012. This could mean that the researcher inputs to software R&D *for that new hardware scale* grew especially quickly during that time, because the scale was previously neglected.
  ○ I already adjusted for this consideration for *AI and Efficiency*, where I ended up on $r = 1$.
  ○ I didn’t adjust for this in *How Fast do Algorithms Improve*, so will make an additional adjustment from $r = 1.6$ to $r = ~1.2$.

• The (very rough) *estimate based on Grace (2013)* would be more affected. It looked at software progress over periods of time when the FLOP/s used by systems increased; if instead the FLOP/s used had remained constant, software progress may have been slower.
  ○ I estimated $r = ~7$, then adjusted down to $r = ~2.5$. I’d now adjust this further to $~r = 2$.

So this mostly affects the estimate of $r$ from Grace (2013), the only one that suggested a software singularity would happen comfortably. The estimates of $r = ~1$ from ImageNet and *How Fast do Algorithms Improve* aren’t affected much.

Paul Christiano and Carl Shulman commissioned [work](#) to investigate this objection. They compared the performance of an old chess algorithm to a new algorithm at both old levels of (physical) FLOP and new levels of FLOP. The old algorithm was 1998 Fritz [black line] and the new algorithm was 2021 Stockfish [blue line].
[x axis shows compute used by a system; y axis shows its Elo rating. Each line corresponds to a different algorithm.]

If the objection is correct, 2021 Stockfish should have a bigger advantage at the new hardware scale (~10,000 kNodes) than the old scale (~100 kNodes).

It's ambiguous whether this is the case. It depends how you measure it.

- The Elo differences between the two algorithms are slightly bigger at the *old* hardware scale. At 100 kNodes, the Elo gap is ~1000, at 10,000 kNodes it's ~800. This suggests algorithmic progress didn't rely on increasing the hardware scale.
- But the efficiency improvements are bigger at the *new* levels of FLOP than at old levels of FLOP. Suppose you ask “how many times fewer FLOP does the new algorithm need to match the performance of the old algorithm?”. 2021 Stockfish needs 100X fewer FLOP to match the performance of 1998 Fritz at 100 kNodes, vs ~500X fewer to match its performance at 10,000 kNodes.\(^{261}\)

---

\(^{261}\) The data also suggests that capability improvements are more significant than efficiency improvements. New algorithms achieve capabilities that would have taken old algorithms >10 OOMs of extra FLOP. By contrast the efficiency improvements are 2 - 3 OOMs. This confirms OpenAI's claim, *above*, that the first time a capability is achieved, the algorithm used is typically *much* more efficient than pre-existing algorithms at achieving that capability.
Overall, it seems plausible that software progress does depend on moving to new hardware scales to some extent. This mostly affects the estimate based on Grace et al, which was already extremely rough. This consideration slightly decreases my probability that a software singularity occurs, down from ~70% to ~65%.

Summing up

I’ve tried to assess the plausibility of a software-only singularity by looking at data about the historical returns to software R&D. I proceeded in a few steps:

- ImageNet data left me thinking that there was ~50% chance of a singularity based on efficiency improvements alone.
- Including significant potential gains from capability improvements increased this to ~85%.
- If a software-only singularity does occur, I guessed it might last for ~5 OOMs.
- Considering two potential bottlenecks, neither of which seemed compelling to me, lowered my estimates somewhat:
  - ~65% chance of a software-only singularity
  - I expect it to last ~2-3 OOMs if it does occur.
- Importantly, even if there is no software-only singularity, software progress might still be extremely rapid just after we fully automate software R&D due to the huge rise in R&D inputs. There could be multiple OOMs of fast progress on a fixed hardware base even if software doublings are slowing down over time. In addition, I expect the quantity of hardware to be increasing rapidly, driving further software progress.

Open questions

I’ve organised these open questions according to which components of the model they’d inform. I highlighted the ones that I thought had the best combination of importance and tractability in yellow.

- **To inform g($) on FLOP globally** - semiconductor production scale up
  - If people were willing to spend $ trillions expanding semiconductor production, how long would it take to double the number of chips produced per year?
    - Via new fabs and more efficient use of existing fabs. *Not* via better chip designs - this falls under R&D scale up.
    - Are there fundamental physical bottlenecks to increasing manufacturing throughput above a certain level? E.g. a certain crystal needs months to be grown and this can’t be expedited.
    - How would the above answers change if there was also abundant specialised cognitive labour (from AIs) to help with the expansion?
    - Use expert networks to speak to someone at TSMC / Intel / Samsung
- To inform \( g(\text{fraction of FLOP on a training run}) \) - prospects for rapid scale up of training runs
  - If you wanted to actually use 30% of global FLOP/s in a training run, how would you do that? What bottlenecks would there be? How long before you can start the training run?
    - What fraction of existing FLOP could you rent? What fraction of new production could you buy without being blocked? How much engineering effort would it take to distribute training over ~1000X more chips than we’ve done to date?
    - Speak to people at Anthropic and OpenAI about the engineering barriers.
    - What is the highest fraction of FLOP/s that could be used in a single training run in various scenarios?
  - Improve our empirical estimates of how the fraction of FLOP on a training run is likely to change over time.
    - What FLOP/s is currently available from all the world’s AI chips?
    - How quickly will the FLOP/s globally and from AI chips grow over the next 10 years?
    - How easy would it be to reappropriate production lines currently producing non-AI chips to make AI chips?
    - How will the FLOP on the largest training run grow over the next 10 years?
- To inform \( g(\text{FLOP/$}) \) - prospects for future hardware progress.
  - Near-term forecasts of FLOP/$ from speaking to industry experts.
    - In a ‘business as usual’ scenario where AI improvements are modest.
    - What happens with hardware if we get AGI in 2030?
      - How many gains are still available from fabless R&D, improving the designs of chips made with existing fab production processes?
      - Once these gains have been taken, what’s the next lowest hanging route to hardware progress?
  - Alternative paradigms: are quantum computing or optical computing plausible over the next few decades? What magnitude of improvement might they bring?
  - If people were willing to spend $ trillions on hardware R&D, how would that affect the rate of progress?
    - How much money could the field usefully absorb?
    - How many people could move in from adjacent fields and usefully contribute?
    - How sharp would the diminishing returns be to increased spending within each year?
    - What are the current bottlenecks to R&D progress, to what extent could they be relieved by more $?
  - If people were willing to spend $ trillions on hardware R&D and there was super-abundant expert cognitive labour (from AIs), how would that affect the rate of progress?
What are the current bottlenecks to R&D progress, to what extent could they be relieved by more $ and abundant cognitive labour?

- **To inform g(2020-FLOP per FLOP) - prospects for future software progress**
  - Gather up to date versions of the data from 'AI and efficiency' paper, for a variety of AI benchmarks.
    - To inform r_software.
  - Do more experiments where you run both old and new hardware using both old and new algorithms. Investigate whether the new algorithms only help with the new hardware, vs whether they help equally with old and new hardware.
    - To inform r_software and whether software progress is dependent on hardware.
  - Think of a new and better way to conceptualize (and ideally quantify) software progress that allows us to achieve new capabilities.
    - The first time a new capability is achieved, the algorithm that achieves it often does so using orders of magnitude less compute than any pre-existing algorithm.
      - E.g. these chess graphs, and section 5.3 of the 'AI and Efficiency' paper.
    - This is at odds with our formalism, in which the compute requirements for new and old capabilities decrease gradually year on year, halving every ~2 years.
    - The challenge here is simply to suggest a new framework for software progress that better captures the nature of software improvements that unlock new capabilities.
      - The new framework may imply, contra Bio Anchors, that we could not have trained AGI with ~1e36 FLOP using 2020-algs.
    - To improve the way I’m modelling software progress.
  - Estimate the correlations between IQ and output on key tasks like R&D.
    - We can combine this with IQ-brain size correlations discussed above.
    - The relationship between brain size and output informs the effective FLOP gap, whether a software singularity is likely to occur, and takeoff speed according to a one-dimensional model of intelligence.
  - Empirically, how ‘jumpy’ is algorithmic progress? What fraction of the total gains happen in unusually large discrete jumps vs normal progress.
    - To inform whether I should put more probability on large discontinuous jumps in capability.
  - During a software-only singularity, might it be possible to avoid retraining each generation of AGIs?
    - What techniques for making AI systems more efficient don’t require retraining from scratch? How big are the efficiency gains from these techniques? How long do they take?
    - If the model size increases significantly, is it possible to avoid retraining the system from scratch (e.g. by initializing the weights of the new larger system using the weights of a smaller system)?
If a new architecture is introduced, is it possible to avoid retraining the system from scratch?

To what extent could ~all AI training be done via online learning, so that precious compute is not “wasted” on training rather than running AGIs?

How can we integrate the answers to these questions in my analysis of whether a software singularity will occur?

- What would ‘automating 20%, 50%, or 80% of software R&D’ look like in practice?
  - Speak to AI researchers about what tasks they perform. Estimate the time spent on each type of task. Describe what it might look like for AI to perform tasks that currently take x% of researchers’ time.
  - What percentage of these tasks could SOTA AI profitably perform today?
  - What percentage of tasks will AI be able to perform with a training run of (e.g.) 1e27 FLOP.
  - To inform whether it will be possible to get large productivity gains from partial software automation in practice.

- How much easier will it be for AI to readily automate a large fraction of AI R&D tasks compared to a large fraction of the broader economy?

- To inform the speed-up from automating AI R&D sooner than the global economy.
  - How much easier will it be for AI to perform all cognitive tasks in AI R&D than all cognitive tasks in the broader economy?

- To inform the size of the effective FLOP gap
  - My research into evidence about the effective FLOP gap was fairly shallow. Two factors in particular could be investigated further.
    - How AI capabilities vary with training FLOP:
      - How does the performance of AI systems vary as we increase the training FLOP 10X - 1000X, but hold algorithms constant? What does this suggest about the increase in training FLOP needed to cross the effective FLOP gap?
      - Are there some domains where it takes significantly more FLOP to train AI than others? E.g. perhaps achieving human level at some band of games takes more FLOP than achieving human level for a comparably narrow band of language tasks.

- How animal capabilities vary with brain size.
  - Pick animals with 3X, 10X, 30X 100X smaller brains than humans. Learn about the cognitive capabilities of these animals.
  - First ask: Could the animal do useful economic tasks (or help with R&D) if they were motivated to help (i.e. if we could perfectly control their second by second desires).
  - Second ask: Could the animal do useful economic tasks (or help with R&D) if their brain had been optimised for this by evolution?
    - This is a weirder counterfactual so harder to think about, but ultimately more relevant to the effective FLOP gap I think.
To the extent the answers are “no, they don’t have the cognitive capabilities to be helpful”, this suggests the effective FLOP gap is small.

- What is the current $ value-add of AI? How is it changes over time, or with model size?
  - Various ways of operationalising this: investment, revenues, effect on GDP.
  - Relevant for when AI will first be capable enough to readily add $trillions / year to GDP.
- Why do MIRI people think there’ll be a rapid (< years) transition from “shallow systems” to “deep systems” where the former aren’t very helpful to science and the latter can ~fully automate science? In the language of this framework, they think the effective FLOP gap is small.
- Dig into AI impacts’ finding that it took decades to cross the human range in chess, Go and checkers.
  - This is in tension with the findings of this report. Some possible resolutions of the tension:
    - The effective FLOP gap is on the high end of my estimates, implying high AGI training requirements.
    - Progress in those games is slower due to slower investment growth and the absence of speed-ups from AI automation.
    - The effective FLOP gap is narrower than in those games, e.g. because “capabilities scale especially quickly in the human range” or “it’s difficult to partially automate jobs”.
  - This is also in tension with the one-dimensional model of takeoff.
  - The first step is probably finding data about how inputs to these domains (compute at training / runtime, software R&D effort) changed while they crossed the human range.

- To inform thinking about bottlenecks.
  - Do bottlenecks raised by economists suggest growth won’t ever accelerate?
  - To what extent do bottlenecks push towards slow takeoff in areas of strategic importance?
  - Critique and improve my analysis of bottlenecks in sections 6 and 9.
  - To what extent is AI progress driven by running big experiments vs software R&D labour? How fast would progress become if we had ~unlimited supply of the latter?
    - Some slightly more fleshed out ideas here.

- Validate this model of takeoff speeds.
  - David Schneider-Joseph makes some suggestions here.
You should read the short and long summaries before reading sections in this document.

6. Bottlenecks from tasks AI can’t perform

I think this section of the long summary summarises most of the important takeaways from sections 5 and 6 in a few pages. I'd only read this section if either:

- you really want to understand the math behind the automation models I'm using further but aren’t familiar enough with growth economics already to read the mathematical description of the Full Takeoff Model.
- you want to know how I’m choosing the value of the “bottlenecking parameters” that control the degree of complementarity between different tasks.

Short summary

This section discusses two types of bottlenecks:

1. **Human bottlenecks.** Sure AIs can perform lots of cognitive tasks and we can run zillions of AIs. But there are some tasks that humans are still needed for and those tasks bottleneck total output. (“Total output” can mean “GDP” or “R&D progress per day.”)

2. **Physical capital bottlenecks.** Sure AGI can perform all cognitive tasks and we can run zillions of disembodied AGIs. But there are still many tasks involving doing things in the physical world, e.g. building fabs and experimenting with chip designs, and those tasks bottleneck total output.

I discuss the strengths of these two bottlenecks in hardware R&D, software R&D, and GDP.

Using a CES task based model, as opposed to the Cobb Douglas model from the last section, provides a convenient way to quantify these bottlenecks. The CES model contains a parameter, \( \rho \), that controls the extent to which total output can be bottlenecked by particular tasks. I estimate values of \( \rho \) for both human bottlenecks and physical capital bottlenecks, and do this separately for hardware R&D, software R&D, and GDP.

This analysis reduces the estimated effects of AI automation, compared to section 5. Section 8 discusses other bottlenecks and delays.
Long summary

In section 5 we modelled software R&D, hardware R&D and Gross World Product (GWP) using a Cobb Douglas task-based model. AI automation was represented via AI performing a growing fraction of tasks in these three areas, with tasks weighted by their relative economic value in 2020.

The Cobb Douglas version of the task-based model omits certain bottlenecks. Suppose we develop AI that fully automates replying to emails, but humans are still needed for other types of work. The Cobb Douglas model implies that, merely by increasing the quantity of email-AIs and holding all else constant, GWP could grow without limit. This is unrealistic: GWP would hit a hard ceiling that cannot be surpassed no matter how many email-AIs you have. To raise output further, you’d need to increase output on tasks other than emails. We’ll see that this dynamic can be modelled by a CES version of the task-based model (explained below).

Essentially, the CES model puts a cap on how much “more output on the same task” can increase software progress per year, hardware progress per year, and amount of goods and services produced per year (i.e. GWP). Total output in these areas are bottlenecked by tasks where output is lower.

This means that having more AIs (at a fixed capability level) doesn’t increase total output past a certain point; the bottleneck can only be removed by increasing output at the tasks AI cannot currently perform.
The blue line shows that “more AIs on the tasks AI can currently perform” cannot increase software inputs, hardware inputs, or GWP past a certain point. Better AI, more human labour, or more physical capital is required to relieve this bottleneck.

This bottleneck might be removed by training more capable AIs that automate new tasks. In the example above, this would involve training AI that can do tasks other than emails. But when bottleneck-ing tasks require physical capital (e.g. physical experimentation), then more capable AIs cannot remove the bottleneck and we’ll have to produce more physical capital.

Below I’ll explain my current guesses about how strong the bottlenecks might be in software R&D, hardware R&D and GWP. In each case I separately discuss bottlenecks that can be
relieved by training better AIs, and ones that can only be relieved by increasing the amount of physical capital.¹

In general, the effect of this analysis is to reduce the estimated effects of AI automation. In software and hardware R&D, AI automation only becomes significant (relative to rising human inputs) when ~40% of cognitive tasks have been automated. (The Cobb Douglas model implied ~25%.)

Again, I’m not aware of a simple, analytically tractable way to calculate takeoff speed metrics given the feedback loops involved here. Instead I simulate the model and do a sensitivity analysis. One result from this is that, including the bottlenecks from this section, AI automation roughly ~halves the time needed to cross the effective FLOP gap.

Basic intuition

A strawman of (part of) the “most important century” series is ‘We’ll build some really awesome AIs. Then those AIs will automate R&D and make economic growth go crazy.’¹

The response from the bottlenecks enthusiast is ‘The pace of economic growth depends more on the stuff we do badly than the stuff we do well. Sure, AI will do loads of things really well. But there are some things AI won’t do well and those things will ultimately place a ceiling on the rate of R&D progress, no matter how amazing your AI is at doing other stuff.’

This same response can be made for the rate of hardware progress, the rate of software progress, the value of goods and services AIs provide directly, AIs contribution to military dominance, etc. In each case, AI might perform loads of relevant tasks really well but not be transformative because of bottlenecks from tasks it cannot perform.

In general, this response is convincing to the extent that i) AI can’t perform some subset of tasks, ii) progress really could be bottlenecked by these tasks despite AI producing high output on the other tasks.

Even if we had zillions of disembodied AGIs, it seems plausible that certain quantities would be bottlenecked by tasks they cannot perform. Some examples:

<table>
<thead>
<tr>
<th>Quantity that is bottlenecked</th>
<th>Cause of bottleneck</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP growth per year</td>
<td>Building many physical copies of newly designed technologies, transporting them around the world, integrating them into workflows.</td>
</tr>
<tr>
<td>Hardware progress</td>
<td>Physical experiments using specialised physical equipment and</td>
</tr>
</tbody>
</table>

¹ I’m including bottlenecks caused by limited supply of human physical labour under the “physical capital bottlenecks” umbrella.
Cobb Douglas vs CES: economic production functions

How can we model bottlenecks?

This section compares two “economic production functions”: formulas for calculating economic output given certain inputs. We’ll see that the Cobb Douglas production function doesn’t incorporate bottlenecks while the CES production function does.

A simple model with two inputs

Let’s start with a simple example with two inputs to production – human labour L and physical capital K – and constant returns to scale.2 (Later we’ll consider the more complicated task-based model.)

The Cobb Douglas formula for output Y is:

\[ Y = K^\alpha L^{1-\alpha} \]

0 < \alpha < 1 gives the fraction of output paid to capital.3 Similarly, labour’s exponent gives the fraction of output paid to labour. Also, the larger an input’s exponent, the more doubling that input increases the output.4 For these two reasons, an input’s exponent can be thought of as quantifying ‘how important’ the input is to production.

In this simple two-input setting, we can ask the following question about bottlenecks: If one input remained fixed while the second increased without limit, would the first input “bottleneck” output? More precisely, would there be a ceiling on output that isn’t exceeded no matter how large the second input becomes?

Let’s imagine holding K fixed while increasing L without limit. What happens to output? Here’s the graph:

---

2 Constant returns to scale means that when you double both inputs, output exactly doubles.
3 In a task-based Cobb Douglas model it also gives the fraction of tasks performed by capital, but it need not have this interpretation. (Not all Cobb Douglas models are task-based.)
4 If you double K, Y increases by a factor of 2^\alpha; so the larger K’s exponent, the larger the effect on Y.
Well, there are diminishing returns to more labour. Each extra worker increases output by less than the previous one. You can see that because the slope is becoming shallower.

However, there is no ceiling on output as labour becomes higher and higher. Even with a fixed amount of capital, if we have enough labour we can achieve any level of output. This can be seen directly from the formula \( Y = K^\alpha L^{1-\alpha} \): each time you double \( L \) you multiply \( Y \) by a factor of \( 2^{1-\alpha} \). If you double \( L \) enough times, \( Y \) can get as high as you like.

It can also be seen from a log-log plot of the same graph.
If we imagine this straight line going on forever to the right (as we increase L), every level of output is eventually reached. So there is no ceiling on total output caused by holding K fixed at 1. In this sense, output cannot by “bottlenecked” by K having a low value.

In this example we held K fixed and increased L; but we’d have got the same result if we had done the reverse.

A popular generalisation of the Cobb Douglas production function is a CES production function. For modelling bottlenecks, we should use a version of the CES production function where labour and capital are complements (as opposed to substitutes).\(^5\) Intuitively, this means that they perform different functions both of which are needed. For example, fast transport requires both vehicles to travel in (capital) and people to drive the vehicles (labour); both the vehicles and people are needed. Henceforth I’ll refer to the version of CES where inputs are complements simply at “CES”.

The formula for the CES production function is hard to interpret, so I won’t include it here. Instead, I will describe the key difference with the Cobb Douglas formula \(Y = K^\alpha L^{1-\alpha}\):

\[ \frac{L}{K} \text{ increases, the exponent on L falls and the exponent on K increases.} \]

Intuitively: the more plentiful an input (compared to other inputs), the less important that input is to production (as quantified by its exponent). As L becomes plentiful, its importance decreases.

\(^5\) Formally, this corresponds to the elasticity of substitution between labour and capital being <1. And (equivalently) to the parameter rho < 0.
And conversely: the more scarce an input (compared to other inputs), the more important that input is to production. As K becomes scarce (compared to L), its importance increases.

For example, suppose you want to transport as many people as possible, and you have 100 cars but only 10 drivers. The people are scarce and so you will value them much more highly. You would rather double the number of drivers than double the number of cars (this corresponds to “drivers” having a higher exponent than “cars” in your formula for “total transportation output”). But if you had 100 drivers and 10 cars the situation would be reversed: you’d rather double the number of cars than drivers (and so you’d have a higher exponent on “cars” than “drivers”). This is a situation where the importance of inputs depends strongly on their scarcity, such that you’d use a CES function rather than a Cobb Douglas. (In Cobb Douglas, you preference for doubling the number of cars vs drivers would not depend on their relative scarcity.)

Let’s ask CES the same question as we asked for Cobb Douglas. If one input remained fixed while the second increased without limit, would the first input “bottleneck” output? More precisely, would there be a ceiling on output that isn’t exceeded no matter how large the second input becomes?

We can plot the same graph as before. We hold K fixed while increasing L without limit, and look at what happens to output Y.

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Alpha gives the exponent on K when L / K = 1. Rho describes the rate at which the exponent on K rises as L / K increases.
Again, there are diminishing returns, but they diminish more steeply. And, importantly, this time there is a hard “bottleneck”. It turns out that output never rises above 4, no matter how large \( L \) becomes. You can see this on a log-log plot of the same graph.  

(Recall that this plot showed a straight line in the Cobb Douglas case.)

\[
\text{CES: } Y \text{ vs } L
\]

\[
K = 1, \alpha = 0.5, \rho = -0.5
\]

Note, in this example the “bottleneck” is capping the GDP that can ever be obtained, because we’re considering the direct production of goods and services (holding the level of technology fixed). But in the case of R&D the “bottleneck” would cap the amount of R&D progress that could be made each year, i.e. it would cap the rate of GDP growth. I’m modelling both kinds of bottlenecks: bottlenecks to how large GDP can become due to AIs providing goods and services while holding the level of technology fixed, and bottlenecks to the rate of (hardware and software) R&D progress.

In the above example, the fixed value of \( K = 1 \) placed a hard ceiling on output of \( Y = 4 \). The exact value of the ceiling depends on the degree of substitutability between \( L \) and \( K \). If they are not at all substitutable, the ceiling is low. If they are somewhat substitutable, the ceiling is higher. (And once they are as substitutable as in Cobb Douglas, there is no ceiling.)

The degree of substitutability is quantified by the variable \( \rho \). The higher \( \rho \), the more substitutable the inputs are. Cobb Douglas corresponds to \( \rho = 0 \), and the bottleneck dynamic occurs if

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7 Mathematically, there is a hard bottleneck because as \( L / K \) tends to infinity, the exponent on \( L \) tends to 0 (while the exponent on \( K \) tends to 1). In this limit, increasing \( L \) makes no difference to output at all.
8 As discussed above, I’m currently not modelling AI automation of generic R&D.
9 In fact, if you are ever so slightly less substitutable than Cobb Douglas, there is a ceiling.
\( \rho < 0. \) We can quantify what the ceiling on output is using \( \rho \). If initially \( Y = 1 \) and capital’s share of output equals \( \alpha \), then as \( L \) tends to infinity output tends to a ceiling of \( (1/\alpha)^{1/\rho} \).

The current capital share is \( \alpha = \sim 0.35 \), which implies the following ceilings on \( Y \) as \( L \) tends to infinity (holding \( K \) fixed).

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<thead>
<tr>
<th>( \rho )</th>
<th>Ceiling on ( Y ) as ( L \to \infty ) (initially ( Y=1 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1.6</td>
</tr>
<tr>
<td>-0.5</td>
<td>6.3</td>
</tr>
<tr>
<td>-0.2</td>
<td>98</td>
</tr>
<tr>
<td>0 (Cobb Douglas)</td>
<td>inf</td>
</tr>
<tr>
<td>&gt; 0 (Labour and capital are substitutes, not complements)</td>
<td>inf</td>
</tr>
</tbody>
</table>

A task-based model

Above we looked at a simple CES model with two inputs and studied its “bottlenecking” dynamic.

A very similar dynamic exists for task-based models. In task based models, each task is considered to be a separate input to production.\(^{11}\) Each task is performed by labour, physical capital or compute.\(^{12}\)

The same dynamic that existed between the two inputs in the previous model exists between every task in this model. This time, \( \rho \) quantifies the degree of substitutability between tasks, rather than between labour and capital. This means that:

- In Cobb Douglas versions of the task-based model, if we hold inputs to some tasks fixed while inputs to other tasks rise without limit then total output rises without limit.
- In the CES versions of the task-based model, if we hold inputs to some tasks fixed while inputs to other tasks rise without limit then there is a ceiling on output.

\(^{10}\) Recall this is the share of output used to rent capital (as opposed to paying wages).

\(^{11}\) We can think of each task \( i \) as having an exponent \( e_i \) which quantifies its importance to production. As before, the exponent \( e \) has two meanings: first, \( e_i \) equals the fraction of output paid to inputs performing task \( i \); second, increasing the output of task \( i \) by 1% increases total output by \( e_i \)%.

\(^{12}\) Recapping section 5: In the FTM a constant fraction of tasks is always performed by capital, a small fraction is initially performed by compute, and the remainder is initially performed by labour. Then over time larger training runs allow more and more tasks to be performed by compute.
The larger the fraction of tasks that rise without limit, the larger the ceiling on output. In fact the formula is the same as in the two input model: if the fraction of tasks that are held fixed equals $\alpha$, and the other tasks rise without limit, then output increases by a factor of $(1/\alpha)^{1/p}$ before hitting the ceiling.

The following table assumes $\rho = -0.5$ (so is CES) and looks at how the ceiling depends on the fraction of tasks that rise without limit (while inputs to the other tasks remain fixed).

<table>
<thead>
<tr>
<th>Fraction of tasks that rise without limit</th>
<th>Ceiling on $Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>1.02</td>
</tr>
<tr>
<td>10%</td>
<td>1.2</td>
</tr>
<tr>
<td>20%</td>
<td>1.6</td>
</tr>
<tr>
<td>50%</td>
<td>4</td>
</tr>
<tr>
<td>80%</td>
<td>25</td>
</tr>
<tr>
<td>90%</td>
<td>100</td>
</tr>
<tr>
<td>99%</td>
<td>10,000</td>
</tr>
<tr>
<td>100%</td>
<td>inf</td>
</tr>
</tbody>
</table>

With this value of $\rho$, increasing output of half the tasks to infinity only raises total output by 4X. Why? Past a certain point, more and better performance of those tasks simply isn’t helpful; we’re bottlenecked by other tasks (whose performance is being held fixed).

I’ve talked here about a ceiling on output GDP, while holding the level of technology fixed. But the same economic models can be applied to R&D. In this case, the ceiling is on the rate of R&D progress, rather than on $Y$.

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13 Tasks are weighted by their 2022 output share: the fraction of 2022 output paid to the inputs that perform that task. For example, if the labour share is 0.65 and workers spend 10% of their time on a particular task, that task’s output share is 0.065. So, the “fraction tasks that rise without limit” is simply shorthand for “the fraction of current output paid to tasks whose inputs rise without limit”. I specify the current year 2022 because the output share of a task increases (decreases) if its inputs become more scarce (plentiful) compared to other tasks.

14 Calcs.

15 Strictly speaking, the ceiling is on the ‘total R&D input’, rather than the ‘rate of progress’. The way the model works is: inputs of labour, capital and compute are used on various tasks → they combine together to make a total R&D input → this total R&D input causes R&D progress. But if total R&D input is constant over time then R&D progress actually slows, due to ideas getting harder to find.
What’s the link with takeoff speeds?

Why am I imagining some tasks rising without limit, while others remain fixed? Because as we approach AGI, the amount of 2020-FLOP will increase very rapidly and so tasks that disembodied AIs can perform will have their inputs increase very rapidly; meanwhile inputs to other tasks continue (at least initially) to change at their current slow pace. The limiting behaviour as AI inputs tend to infinity sheds is relevant to what will happen as the AI inputs become extremely large.

If we make an assumption about the total fraction of tasks that AIs can perform in a given domain, and an assumption about \( \rho \), CES models allow us to deduce how much total output of that domain will increase as AI inputs tend towards infinity. The two key domains are “GDP, holding the level of technology fixed” and “R&D progress per year”. The second domain roughly translates to “rate of GDP growth” as R&D drives technological progress and thus GDP growth.\(^\text{16}\)

So in this CES framework, the strength of the bottlenecks on GDP and R&D progress depend on the fraction of tasks performed by AI and on \( \rho \).

What fraction of tasks might disembodied AI eventually be able to perform? Earlier, I suggested that AGI might perform \( \sim 50\% \) of tasks in goods and services production (compared to a labour share of 65% that includes physical labour), \( \sim 70\% \) of the tasks in hardware R&D and \( \sim 100\% \) of the tasks in software R&D. The uncertainties are large because i) I haven’t done more than a quick google to pin down these parameters, and ii) it’s hard to separate the contribution of “cognitive labour” from that “total labour including manual labour”.

What evidence do we have about \( \rho \)?

There are three buckets of evidence that I’m aware of.

**Firstly, there are empirical studies of the substitutability between labour and capital** in particular industries and the economy as a whole. I have not looked into any of these. I asked Phil Trammell for his opinion and have pasted his reply in a footnote.\(^\text{17}\) Apparently, the standard

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\(^{16}\) Though the *Full Takeoff Model doesn’t model AI automation of generic R&D*, only hardware and software R&D.

\(^{17}\) Phil says: “I think a standard central guess for the elasticity of substitution between labor and capital is 2/3, which would make \( \varphi = -1/2 \). I don’t remember what papers give what numbers, but here are some sources:

- Oberfield and Raval (2014): Estimate \( \varphi \) from data on how individual manufacturing plants respond to changes in wages and capital rents (i.e. interest rates).
- A bunch of studies cited in the intro of *Jones (2003)*: Antras (2004), Hammermesh (1993), Krusell, Ohanian, Rios-Rull and Violante (2000), Caselli and Coleman (2000). I’m less familiar with them, but they use a variety of methods, e.g. international comparisons of K/Y vs. capital share.
value used based on this evidence is $\rho = -0.5$. (Recall, $\rho = 0$ is Cobb Douglas with no bottlenecks, and the lower $\rho$ the stronger the bottleneck. So $\rho = -0.5$ corresponds to a fairly mild bottleneck.)

In addition to the usual uncertainties with this type of empirical work,\textsuperscript{18} there are two additional uncertainties:

- **Extrapolating to a scenario with huge AI inputs.** The evidence for these studies is the minor variations in (e.g.) capital intensity that have occurred in recent decades, but this is very different to a scenario in which AI inputs are doubling every 6 months or faster. To make predictions about the AI scenario, we essentially have to extrapolate out the evidence from minor variations between different inputs to a scenario with huge variations between different inputs. I basically don’t trust these studies at all to make good predictions about this type of scenario.

- **Extrapolating from “labour tasks vs capital tasks” to other “AI vs non-AI”.** From the perspective of the task-based model, these studies estimate the substitutability between ‘tasks performed by labour’ and ‘tasks performed by capital’. But, conceivably, this might be different from the substitutability between cognitive tasks and non-cognitive tasks (which include both tasks done by both capital and physical labour). In particular, you might think physical labour is better able to substitute for capital than pure cognitive labour, suggesting a lower value of $\rho$ for cognitive vs non-cognitive tasks. And this (potentially) lower value is the one we’re interested in, as disembodied AI will only automate cognitive tasks.

One last thing about this first bucket of evidence. Longer run studies tend to find higher values of $\rho$ – sometimes as high as 0 (which is the Cobb Douglas case).\textsuperscript{19} Jones (2003) ventures an interesting hypothesis as to why. Suppose we have a sudden influx of AIs. Jones’ hypothesis is that in the short run we can’t effectively use all these AIs in production without getting bottlenecked by our lack of (e.g.) physical capital; so output is bottlenecked and $\rho$ is low. But in the longer run we invent new production processes that use our new balance of inputs more

\textbf{Chirinko and Mallick (2017):} Responding to some of the criticisms about short- vs. long-run $\rho$ (as we’ve discussed coming from Piketty), they look at a long-run data set and see whether periods of growth in K/Y (in the US as a whole or by industry) were associated with subsequent increases or decreases in the capital share. (Though these long-run correlations are tricky.) They estimate $\rho$ a bit closer to 0 after allowing for long-run adjustments, but still negative.

None of these are for task-based models in particular, as far as I can recall, but presumably the economy-wide (as opposed to firm- or industry-specific) estimates should be the same whether or not you write down your model as just being CES overall or broken up into a bunch of tasks."

\textsuperscript{18} E.g. “Were the statistical techniques used successfully able to pin down the parameter in the narrow setting of the study, given that many potentially-relevant factors that are hard to control for”, “Do the results from the narrow setting generalise to the economy more broadly?”.\textsuperscript{19} [Rabbit hole-y fn.] For example, the simple fact that L/K has decreased significantly over recent decades, yet the capital share has stayed ~constant, naively suggests that rho = ~0. From the perspective of a CES task based model, however, this isn’t convincing because we have automated tasks over recent decades and the CES model predicts that this increases the capital share. This automation can counteract the decrease in L/K.
effectively, e.g. finding a way to produce the things we need using lots of AIs and limited physical capital; this relieves the bottleneck and $\rho$ becomes higher. On this view, bottlenecks are a short-term phenomenon and Cobb Douglas, meaning $\rho = 0$ and no bottlenecks, is accurate over longer timescales.\textsuperscript{20}

Secondly, there is a variety of evidence suggesting that $\rho < 0$. In particular, there’s strong evidence that “when a task in the economy becomes more productive (relative to other tasks), its fraction of GDP declines”. This implies that $\rho < 0$.\textsuperscript{21} (Why? Recall that the the fraction of GDP paid to perform a task is quantified by the task’s exponent, task exponents are constant in Cobb Douglas ($\rho = 0$), but when $\rho < 0$ the exponent declines as the task becomes more abundant.)

Examples:

- **Agriculture** used to be a large fraction of GDP (hundreds of years ago). As its productivity and output surged, its share of GDP fell. It’s now 5\% of US GDP. Past a certain point, we just don’t want more food. As it becomes more abundant, we spend a lower fraction of our resources on it.

- **Manufacturing** industries have on average enjoyed larger productivity gains than services industries. Yet their share of GDP has declined across the same period. Again, as manufactured goods become more plentiful, we spend a lower fraction of our resources on them.

- **FLOP**. Tasks that people used to pay lots of $ for computers to perform are now dirt cheap. I expect that the fraction of GDP spent on these tasks has fallen dramatically, even as our ability to perform them productivity has skyrocketed (with rising FLOP/$).

One confusing thing with interpreting this second bucket of evidence, that I don’t feel satisfied with, is teasing apart demand and supply effects. The above examples seem to relate to demand: as we produce more of product X, we spend a smaller fraction of GDP on it. But, especially when imagining AI doing R&D or increasing military power, I’m more concerned with supply. That is: if we want more of Y (GDP / R&D progress / military power), does the importance of sub-tasks to achieving Y fall as we perform those tasks more/better?\textsuperscript{22} It seems possible that, despite these demand effects, there are some tasks for accelerating R&D, or gaining power, that would remain important even as AI performs them much more/better.

\textsuperscript{20} A temporary bottleneck could still prevent fast takeoff, if it takes decades for it to be removed. The key question is: What ‘causes’ the bottleneck to be removed over time? If it is cognitive labour designing new production processes, then AIs can do that quickly and remove the bottleneck to their own economic impact. If it is instead schlep integrating AIs in the economy, then AIs may not be able to remove their own bottleneck.

\textsuperscript{21} It may be possible to use this kind of data to estimate the exact value of rho, rather than merely its sign. I’m not aware of studies of this kind.

\textsuperscript{22} This coincides with the demand effect when we define Y as “satisfying people’s demands for goods and services”.

Overall, I think this bucket gives good evidence that $\rho$ is noticeably below 0 for GDP, i.e. when considering different tasks for producing goods and services that people want to buy. I think it also gives some reason to believe that “as AI inputs rise dramatically, the tasks that they can perform will become less important to R&D, military power, and other strategic domains”.

The third bucket of ‘evidence’ is simply doing inside-view thought experiments about what you think would happen in a world with zillions of AGIs working on (e.g.) hardware R&D. How much more quickly could they improve chip designs than we are currently, despite having access to the same fixed supply of physical machinery to use for experiments? If you think that hardware progress would be 100X its current pace, you can use this to “back out” a value of $\rho$ consistent with that. This type of thought experiment gets at $\rho$ for cognitive tasks vs non-cognitive tasks. Or you could run the thought experiment for software R&D or GDP (imagining zillions of AGIs producing goods and services, holding fixed the physical machinery).

[How could you “back this out”? Recall that if AI can perform a fraction $1 - \alpha$ of value-weighted tasks then, as AI inputs tend to infinity, output increases by a factor of $(1/\alpha)^{1-\rho}$. So if you think hardware progress would be 100X its current pace in this hypothetical, you’re estimating that $(1/\alpha)^{1-\rho}$ equals 100. If you also have an estimate of $\alpha$ you can “back out” an estimate of $\rho$.]

Doing this kind of inside-view thought experiment gets into lots of tricky issues like “Could you replace physical experiments with simulations?” and “How many experiments would be needed for a smart enough team of AIs to discover nanotech and use it to design better chips?”. These questions are, I think, worthy of much more investigation. It would be useful to think through specific candidate bottlenecks concretely and assess how much they would slow down progress.

This third bucket of ‘evidence’ leads me, at least in the case of hardware R&D, to higher estimates of $\rho$ than the first bucket. If $\rho = -0.5$ and $\alpha = 0.3$ (as I suggested for hardware R&D), then even zillions of AGIs would only increase the pace of hardware progress by ~10X. But with billions of AGIs thinking 1000X as fast and optimising every experiment, I think progress could be at least 20X quicker than today, plausibly 100X. If $\alpha = 0.3$, a 100X speed up implies $\rho = -0.25$. I expect some people to favour larger numbers still. Very large numbers would favour choosing a value of $\rho$ very close to 0 (but still negative), which would approximate Cobb Douglas ($\rho = 0$).

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23 By “noticeably below 0” I mean that using Cobb Douglas, where $\rho = 0$, would not give approximately correct predictions in practice. E.g. I think the evidence here suggests $\rho < -0.1$.
24 Or you could do the same thought experiment but have the disembodied AGIs do generic R&D or directly provide goods and services.
25 If your multiplier on output (from AI inputs going to infinity) was S, $\rho = \ln(\alpha) / \ln(S)$.
26 Recall that alpha gives the fraction of value-weighted tasks performed by physical capital in 2020; it is the same as the capital share of R&D.
27 If $\rho$ isn’t negative, then the rate of R&D progress goes to infinity as the number of AIs doing it tends to infinity.
My instinct is similar for generic R&D, and for military power. Zillions of AGIs thinking 1000X as fast could, despite a fixed stock of physical equipment, increase your military power by more than 10X, plausible by 100X. This implies $\rho > -0.5$. Again, these are simply my instincts and this third bucket really deserves a lot of further investigation.

**Summing up**

I expect AI will increase output on certain tasks by many orders of magnitude compared to today, while not increasing output on other tasks much at all. How much might total output (‘GDP holding technology fixed’ / ‘the rate of R&D progress’) increase in this scenario before it is capped by the tasks not performed by AI?

The task based model gives a tool for answering this.

When $\rho = 0$ (Cobb Douglas) there is no cap on output, output can increase without limit if you have enough AIs. But when $\rho < 0$, output is capped below a ceiling that depends on the fraction of tasks performed by AI and $\rho$.

All three buckets of evidence about $\rho$ suggest $\rho < 0$. The first bucket has a central estimate of -0.5, but closer to 0 over longer time horizons. The second bucket doesn’t (to my knowledge) provide precise estimates of $\rho$, but suggests $\rho < 0$. The final bucket is fraught and underdeveloped. To my mind, it suggests that, at least for R&D and military power, $\rho > -0.5$ and perhaps very close to 0.

The rest of section 6 discusses how to model bottlenecks specific to software R&D, hardware R&D, and GDP.

**2020-FLOP per FLOP**

I’ll discuss two bottlenecks:

1. **Human bottlenecks.** When AI has automated many but not all cognitive tasks, progress may be bottlenecked by the non-automated tasks still performed by humans.

2. **Physical FLOP bottlenecks.** When AI has automated all cognitive tasks, progress may still be bottlenecked by the time it takes to either i) do large experiments testing new algorithms or ii) train new (and better) AIs.

**Human bottlenecks**

In section 5 I calculated that, when AI had automated a fraction $f$ of tasks, software inputs would grow at $f \cdot g(2020\text{-FLOP})$. 
But this calculation used a Cobb Douglas model that assumed that tasks remain equally
important even when AIs perform them much more/better. In a CES model with $\rho < 0$, the tasks
done by AIs will become less important and so software inputs will rise more slowly.

What value of $\rho$ should we use? In this setting we really care about the degree of substitutability
between different cognitive tasks. (Recall that all the tasks for software R&D are cognitive, in my
model.) Unfortunately, the evidence discussed above is not directly relevant to this.\footnote{In particular, bucket 1 related to labour tasks vs capital tasks, bucket 3 to cognitive vs non-cognitive
tasks, and bucket 2 was about the substitutability of different sectors of the economy for GDP. None of
these really speak closely to the substitutability between different cognitive tasks.}

If I think about the tasks involved in software R&D, it does seem like there will be bottlenecks. In
particular, I imagine AIs automating some subset of tasks, and having the ability to do these
tasks to an arbitrarily high speed and quality. It doesn’t seem like the rate of software progress
increases without limit; instead human tasks bottleneck progress.

- Suppose AI automates the process of implementing new algorithms (given a description
  in natural language or maths), but humans still have to invent these algorithms. If the
  number and quality of AIs able to do this grow without limit, the rate of software progress
  is still bottlenecked by the number of humans available to invent new algorithms.
- And $vice versa$. Suppose AI automates the invention of new algorithms, but humans still
  have to implement them. This time it feels like the rate of software progress might
  increase more before hitting a ceiling, as AIs could invent new algorithms faster and
  invent higher quality algorithms. But eventually progress would be bottlenecked by the
  time taken for humans to implement the best algorithms that AIs could invent with the
  information available to them.
- More generally, it feels to me like R&D progress often relies on many successive
cognitive tasks being completed, all of which are necessary to make a unit of progress.
  (E.g. think of a hypothesis, write it down, implement it, test it, interpret the results, iterate,
  write up the results.) This lends itself to bottlenecks as any task can hold up progress.

Overall, I’m really not sure what value of $\rho$ to use here. I’m going with $\rho = -0.5$ as a central
estimate; but am open to as high as -0.2 or as low as -2. This is on the lower end suggested by
the evidence discussed previously, because it does seem likely to me that software R&D will
involve some non-negligible bottlenecks if only some tasks are automated. [Interested in
thoughts of ppl with more context on this type of work.]

In section 5, I suggested that AI automation would become more important than rising human
inputs when around 20% of cognitive tasks were automated. How does this change when we
move from Cobb Douglas to $\rho = -0.5$? The analysis in CES is complicated, but after playing
around with the model a little with, my estimate is that AI automation becomes more important
than rising human inputs only when around $\sim 40\%$ of cognitive tasks are automated.
Another way to ‘grok’ the effects of partial software automation, that’s probably more useful, is to ask: if we automated x% of software R&D tasks, and AI inputs to those tasks were very high\(^{29}\) (such that we hit the human bottleneck\(^{30}\)), how many times faster would software progress be (compared to if we automated 0% of tasks)?

The following table shows this for three values of \(\rho\).\(^{31}\)

<table>
<thead>
<tr>
<th>Fraction of software R&amp;D tasks automated with very high AI inputs</th>
<th>How many times faster could software progress be (compared to automating 0% of tasks)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho = -1)</td>
<td>(\rho = -0.5)</td>
</tr>
<tr>
<td>10%</td>
<td>1.1</td>
</tr>
<tr>
<td>20%</td>
<td>1.25</td>
</tr>
<tr>
<td>50%</td>
<td>2</td>
</tr>
<tr>
<td>80%</td>
<td>5</td>
</tr>
<tr>
<td>90%</td>
<td>10</td>
</tr>
</tbody>
</table>

Physical FLOP bottlenecks: experiments

The human bottleneck discussed above stops applying once AI can perform all cognitive tasks (i.e. once we have AGI). At this point, it might seem like AI can perform all the work necessary for software R&D, and so there will be no more bottlenecks. However, one important part of AI software R&D today is simply doing experiments and seeing what works. While researchers may have reasons to think a new algorithm will have a particular effect (e.g. based on a priori

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\(^{29}\) In practice, I expect the actual rate of progress will not be too far from these limits. Once we have enough compute to train an AI that automates x% of tasks, we typically already have enough compute to run many millions or billions of copies. This is especially true if medium or long horizons are required for training. There are a fairly small number of human researchers doing software R&D (currently < 100,000), so it should be possible to run enough AIs that AI per-task inputs are much higher (~100X) than human per-task inputs. If it’s possible, it will likely happen because after "wake up" the demand for increased inputs to software R&D will be high. If AI per-task inputs are ~100X higher than human inputs then, for low values of \(\rho\) (\(\rho < -0.5\)), overall software progress is close to the maximum at the bottleneck.

\(^{30}\) As described above, as AI inputs tend towards infinity, the rate of R&D progress approaches a ceiling due to the limited human inputs to the remaining tasks. This ceiling can be expressed multiple higher than if humans performed all the tasks

\(^{31}\) A hacky way to generate an equivalent table for hardware R&D would be to divide the percentages in the left column by 0.7, i.e. multiply them by 1.4. (0.7 is the fraction of hardware R&D tasks done by cognitive labour.) This hack is accurate if, for hardware R&D, \(\rho = -0.5\) between different cognitive tasks and \(\rho = 0\) between physical capital and cognitive labour. Below I settle on -0.25 for the latter quantity, which reduces the impact of automation.

\(^{32}\) I think we’re less likely to approach these limits than for the other two columns in this table, even if \(\rho\) has this value. This is because we’d need to run many more AIs on each task to approach the human bottleneck.
mathematical arguments or intuitions developed from previous experiments), these results have to be demonstrated in practice (e.g. by achieving good performance on pre-existing benchmarks) before new algorithms are published. And it’s not always possible to predict the results in advance.\textsuperscript{33}

Here’s a simple model in which this bottlenecks software progress.

- Software R&D has two stages: Design and Test. In the Design stage new algorithms are designed and implemented; in the Test stage their performance is tested. The test results are used to design the next algorithms.
- The speed of Design is proportional to 2020-FLOP/s used for software R&D (as in our model from the last section).
- The speed of Test is proportional to the physical FLOP/s used for testing.\textsuperscript{34}

Suppose that, absent the Test stage, there would be a software-only singularity, with each doubling of 2020-FLOP per FLOP faster than the last even with a fixed amount of physical FLOP/s. AIs designing better AIs, which design better AIs, in an explosive loop. The Test stage would bottleneck this process as it cannot be sped up while using a fixed amount of physical FLOP/s.

This bottleneck may prevent a software singularity happening, or may place a limit on how fast software doublings can become, during a temporary\textsuperscript{35} software-only singularity.

A related possibility is that historical software progress has relied on our algorithms using increasingly large amounts of compute during training and runtime, as new hardware scales opened up the possibility for new types of software innovations. This would have a similar implication: software progress would be limited by the rate at which we accumulate physical FLOP/s. More.

The FTM assumes the computational experiments have a share of software R&D of 30%, with 70% going to cognitive tasks. The 30% matches our previous assumption about capital’s share of hardware R&D. I use $\rho = -0.01$ to approximate a Cobb Douglas production function without hard bottlenecks, because I don’t expect a fixed supply of physical FLOp for experiments to put a hard cap on the rate of software progress.

\textsuperscript{33} E.g. it’s my impression that some people were surprised by how well simple algorithms have been able to solve certain tasks once they have enough data, compared to more complex algorithms that were specialised to perform that very task.

\textsuperscript{34} Why can’t the AIs design more efficient tests? In some cases they may be able to. E.g. if they design a new algorithm to do the same task faster, their tests may get faster. But in some cases this may not be possible. E.g. if they design a new algorithm that uses the same physical FLOP/s but is smarter. The new algorithm uses just as much physical FLOP/s as the previous, so takes just as long to test. It’s this second kind of test I’m saying could be a bottleneck.

\textsuperscript{35} By “temporary software singularity” I mean that after a while the software doublings slow down over time: each takes longer to happen than the last. More specifically: there are $\geq 2$ doublings of 2020-FLOP per FLOP, each (at least slightly) faster than the last and happening on a ~fixed base of physical FLOP/s, and then doublings slow down over time.
This concludes discussion of the bottlenecks to software R&D; the next section briefly examines the bottlenecks for hardware R&D.

**FLOP/$**

I think there are three main bottlenecks to hardware R&D: human bottlenecks, physical capital bottlenecks and delays to printing new chips. I discuss each in turn.

**Human bottlenecks**

These are bottlenecks from unautomated cognitive tasks that must still be performed by a limited number of humans. They occur when AI has automated some but not all cognitive tasks.

This can be modelled just as in the case of software R&D. Again, my best guess central estimate is to use $\rho = -0.5$ as the parameter describing substitutability between different cognitive tasks.\(^{36}\) [This table] describes the bottleneck this places on overall cognitive output, if AI automates various fractions of cognitive tasks.

**Bottlenecks from physical capital.**

This bottleneck becomes significant once AI has significantly increased total output on cognitive tasks for hardware R&D. This happens after full automation of cognitive tasks but also before this point, when AI automates a large enough fraction of cognitive tasks that overall cognitive output has become very large. (E.g. if AI automates 90% of cognitive tasks, with all humans concentrated on the remaining 10%, this boosts total cognitive output by at least 10X, probably more due to more/better performance of the other 90% by AIs.)

Note, in the FTM “physical capital” includes the non-cognitive elements of manual labour. (E.g. for a plumber there’s the cognitive task of deciding how to solve the problem step by step, the cognitive task of sending instructions for implementing this solving, then the “pure manual labour” task of actually physically solving the problem. A disembodied AI might tele-advice a plumber, or teleoperate a robot-plumber, but it can’t do the “pure manual labour” element of the task of plumbing.)

This is about the substitutability between cognitive labour and physical capital.\(^{37}\) This could be different to that between different cognitive tasks. So while I’ll use $\rho = -0.5$ for the substitutability between cognitive tasks, I’ll choose another value of $\rho$ for the substitutability between cognitive labour and physical capital.

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\(^{36}\) In particular, the FTM calculates the combined output of all cognitive tasks as a CES function of the inputs to each cognitive task, with rho < -0.

\(^{37}\) Or, in the language of the task-based model, substitutability between the collection of tasks performed by cognitive labour (AI or human) and the collection of tasks performed by physical capital.
What should we think about this $\rho$?

Above I discussed the third ‘bucket of evidence’ about bottlenecks: if there were zillions of AGIs doing hardware R&D, but they had to use current physical machinery for experiments, how much faster could they advance hardware R&D compared to the current rate?\(^{38}\) The following table shows how your answer determines $\rho$, assuming that the current capital share of R&D is 30%.\(^{39}\)

<table>
<thead>
<tr>
<th>How much could zillions of AGIs accelerate hardware progress with current physical capital?</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-0.5</td>
</tr>
<tr>
<td>20</td>
<td>-0.4</td>
</tr>
<tr>
<td>50</td>
<td>-0.3</td>
</tr>
<tr>
<td>100</td>
<td>-0.25</td>
</tr>
<tr>
<td>1000</td>
<td>-0.17</td>
</tr>
<tr>
<td>10,000</td>
<td>-0.13</td>
</tr>
</tbody>
</table>

Based on this table, my very tentative central estimate of $\rho$ would be -0.25, with a conservative guess at -0.5 and an aggressive guess of -0.17. This is broadly consistent with evidence from bucket 2 (which supported $\rho < 0$ but didn’t provide a specific estimate) and bucket 1 (which suggested $\rho = -0.5$ for certain industries but $\rho$ closer to 0 over longer periods of time). I think the best next steps on making this less made-up would be to look more concretely at how hardware R&D works and what necessary bottlenecks are imposed by the need to conduct physical experiments and wait for the results.

The following graph shows the behaviour of my central estimate and my aggressive and conservative guesses. In particular, it shows how the pace of R&D progress varies with the total cognitive inputs to R&D.

---

\(^{38}\) This leaves it ambiguous whether the AGIs have access to unlimited compute for running simulations. In the FTM, the potential importance of simulations for hardware R&D is not modelled. In practice, then around and shortly after AGI there will be a fair bit of computation available for simulations like this. (They’ll be at least the compute needed to train AGI, and there will be strong incentives to use compute for hardware R&D.) But there won’t be ~infinite computation sitting around. So probably the best thing to imagine for this thought experiment, is “you have a decent amount of compute for simulations, but not an insane amount so you can’t just brute force things”.

\(^{39}\) Calcs.
The x axis gives total cognitive input to R&D, varying from today's inputs on the far left to 1 billion times today's inputs (“1e9”). One way to imagine increasing cognitive inputs by (e.g.) 10X is to imagine all the current R&D researchers think 10X as quickly. The y axis shows how much faster R&D progress is, compared with if we were still using today's cognitive inputs.

So, for example, the yellow line suggests that if today's hardware R&D researchers could think 10X as quickly (but had access to the same physical capital) hardware progress would be 5X faster. If they could think 1000X as quickly, progress would be 50X faster. And if they could think 10 million times as quickly (1e7), progress would be 500X faster.

I include these graphs to give readers a sense of how assumptions about $\rho$ translate into predictions about the pace of R&D progress as we ramp up cognitive inputs (via AI) but leave physical inputs constant.

One last thing on bottlenecks from physical capital. I’ve focussed on how severe the bottlenecks here might be if we have huge AI inputs but the amount of physical capital is growing much more slowly by comparison. But one way to avoid these bottlenecks is to very quickly increase the amount of physical capital (e.g. by quickly building more specialised machinery and robots

Calcs.

You shouldn’t imagine doubling the number of researchers as i) there are “stepping on toes” effects as 2X as many researchers are harder to coordinate (e.g. more duplicated research) and so don’t produce 2X cognitive output, and ii) you would be duplicating the number of physical bodies that can run experiments (which should be held constant).
to conduct experiments). I haven’t thought much about this side of the equation. Here are some very brief and incomplete thoughts:

- A crucial input into making more physical capital is physical capital itself. Most machines are made with the help of other machines. While plentiful cognitive labour can optimise the process of constructing physical capital, there will be limits to this. The process will be bottlenecked by the already-existing physical capital and physical labour. This makes it at least plausible that the bottleneck here could be slow to resolve.

- In the case of hardware R&D, it may be possible to quickly reappropriate physical capital from other areas of the economy. (Either machines used directly on hardware R&D, or machines that can make machines used in hardware R&D.)
  - E.g. Carl believes that car factories currently could be refitted to produce robots without too much difficulty. Disembodied AIs could control these robots remotely to do experiments for R&D.
  - Of course, this is only possible if AIs can quickly design sufficiently good robots.

- Given the fast growth of 2020-FLOP around AGI, it does seem very likely that physical capital will be growing much slower and therefore be a bottleneck.

- I don’t trust the FTM’s assumptions about physical capital growth around and after AGI, and think they probably underestimate the growth of the relevant kinds of physical capital. More.

Lags from hardware R&D to having new chips.

I have already briefly discussed this bottleneck. The (physical) FLOP/s that can be done by the global stock of chips is the result of many years of production. Correspondingly, it would take several years to produce enough cutting edge chips to match the current global stock of FLOP/s. One guess is that it would take ~3 years.\(^\text{42}\)

Suppose you design an improved chip that does 2X more FLOP/s than the current SOTA chip. Even if you start manufacturing it straight away, it will be years before you have significantly affected the FLOP/s that can be done by chips globally. This creates a lag from “hardware progress” to “more (physical) FLOP”.

If you need to build a new fab, there will be an additional delay before you even start manufacturing the new chips.

The FTM models both these delays (more). Huge demand for AI chips, and AI automation, may reduce both these lags; it would be good to investigate how much they could be reduced with sufficient demand and sufficient cognitive labour.

\(^\text{42}\) Data from Bio Anchors suggests FLOP/s per $ grew at 23% from 2008 - 2018. If $ spent on chips each year grew at 10%, then FLOP/s grew at a rate of 33%. It turns out that, if the FLOP/s from chips produced each year is growing at 33% then it takes \(1 / 33\% = 3\) years of current production to produce as much as is global stock.
GWP

Recall that I'm assuming the effect of AI automation on $ on FLOP mirrors its effect on GWP. If GWP growth is boosted from 3% to 5%, g($ on FLOP) is boosted from (e.g.) 22% to 24%.

I think there are three main bottlenecks to GWP: human bottlenecks, physical capital bottlenecks and other barriers to AI’s economic impact. I discuss each in turn.

Human bottlenecks

This is the same as for software and hardware R&D. When AI has automated some but not all cognitive tasks, the tasks still done by humans may bottleneck progress. As before, I'm modelling this using a CES task-based model with $\rho = -0.5$. This table describes the bottleneck this places on overall cognitive output, if AI automates various fractions of cognitive tasks.

Physical capital bottlenecks

Conceptually, this is the same bottleneck that I discussed above in relation to hardware R&D. In the case of hardware R&D, my best guess was $\rho = -0.25$ for the substitutability between cognitive labour and physical capital (vs $\rho = -0.5$ for the substitutability between different cognitive tasks).

There are a few reasons why I think the physical capital bottleneck will apply more strongly for GWP than for hardware R&D:

- Evidence from bucket 1 (empirical measurements of the substitutability between capital and labour) and bucket 2 (the observation that when economic sectors become more productive their share of GDP falls) is more relevant to GWP than to hardware R&D. They imply stronger bottlenecks (lower $\rho$) than $\rho = -0.25$ (our choice for hardware R&D).
  - Bucket 1 suggests $\rho = -0.5$ in the short run, higher in the longrun.
  - Bucket 2 suggests $\rho$ noticeably lower than 0, but doesn’t give a specific estimate (to my knowledge).
- Evidence from bucket 3 (thoughts experiments about worlds with super-abundant disembodied AGIs) suggests a stronger bottleneck for GWP than for hardware R&D.
  - For hardware R&D, it seemed plausible that zillions of AGIs could optimise the experiments done enough to speed up hardware progress by 100X.
  - With GWP, it seems to me like people’s willingness to pay for products that cognitive labour can provide would dry up long before their incomes increased by 100X.
  - Yes people would enjoy abundant personalised entertainment, medical advice, any other expert advice, companionship, education, and other
‘cognitive products’. (We’re putting aside the benefits of faster general technological progress here; just focussing on the role of abundant AGIs in directly delivering goods and services.)

- But many things that people want involve interaction with physical objects: e.g. housing, travel, food, material goods. Without improvements in these things, I’d guess that the income gains from cognitive products would be limited at a low multiple of their current income.

- My strong suspicion is that the economists who reviewed my GWP report would agree.

- If the fraction of value-weighted tasks performed by cognitive labour is 0.5, as I assumed earlier, then \( \rho = -0.5 \) implies this multiple equals 4.\(^{43}\) This does seem too a little low to me; maybe I’d have said \(~10\), which corresponds to \( \rho = -0.3 \).\(^{44}\)

- If AIs created really awesome virtual worlds, and people were happy to live in them, then perhaps the gains could be much much bigger. But this requires technological progress which I’m putting aside.

Whatever value of \( \rho \) we choose for GWP will also apply to $ on FLOP, as I’m assuming AI automation affects both in the same way. Recall, I’m interpreting “$ on FLOP” as roughly meaning “the number of chips”. So the thought experiment here is: with constant physical equipment how much could zillions of AGIs increase the number of chips produced (holding fixed the chip quality)? Again, a fairly low multiple seems reasonable here.

Overall, my best guess here would be \( \rho = -0.4 \), for the substitutability between cognitive labour and physical capital in the production of goods and services.

Other barriers to AI’s economic impact

There are other reasons why abundant disembodied AGIs might (at least initially) have only a limited economic impact.

- Regulations might prevent or delay AI rolling out into the economy.
- Incumbent labourers might resist being replaced by AI workers.
- People might distrust AI workers.
- … other things

I’m not modelling these bottlenecks. This is a reason to expect the economic impacts of AI systems to be delayed relative to my forecasts. This could make takeoff faster or slower, depending on the takeoff metric used and various empirical factors. I discuss this more in section 8.

\(^{43}\) Recall that the ceiling on output equals \((1 / \alpha)^{\lambda (1 / -\rho)}\). In this case \( \alpha = 0.5 \) and \( \rho = -0.5 \). So the ceiling equals \( 2^{0.2} = 4 \).

\(^{44}\) Rho = \( \ln(\alpha) / \ln(\text{ceiling}) = \ln(0.5) / \ln(10) = 0.3 \).
I expect these bottlenecks to apply more strongly for jobs where AIs would be directly providing goods and services to humans, compared with R&D. My impression is that R&D is harder to monitor and regulate, and not interacting directly with human customers would remove certain problems. This is why I haven’t highlighted this type of bottleneck for R&D.

Similarly, I expect these bottlenecks to apply less strongly to the early stages of the production process (e.g. extracting raw materials, combining them together to make intermediate products, combining and transporting intermediate products). This includes most of the stages in the production of computer chips, e.g. building fabs, making the inputs needed by fabs, making computer chips, transporting them.

This suggests that these bottlenecks may apply less strongly to $ on FLOP (~“number of chips”) than to GWP. So this model may be more trustworthy for $ on FLOP (which feeds into forecasts of AI capabilities via increasing the largest training run) than for GWP (which is a measure of AI impact). 45

**Summing up**

Even if AI inputs are soaring (due to training better systems and running many more copies of each), output can be bottlenecked by slower-growing non-AI inputs like human workers and physical capital.

This is true in software R&D, hardware R&D, and in GWP. In section 5 I modelled each of these three processes using a Cobb Douglas task-based model. To incorporate bottlenecks, I’m replacing these with CES task-based models. The parameter $\rho$ controls the substitutability between different tasks and thus the strength of the bottleneck. In software R&D, hardware R&D, and GWP I use $\rho = -0.5$ to represent the human bottleneck from unautomated cognitive tasks. In hardware R&D I use $\rho = -0.25$ as my best guess for the bottleneck from physical capital; in GWP I use $\rho = -0.4$ for this bottleneck.

The exact implications for takeoff speed metrics are hard to calculate analytically. They depend on the size of the effective FLOP gap, AGI’s training and runtime requirements, and the speed at which human investments ramp-up. To address this shortcoming, the next section looks at takeoff dynamics in some example scenarios and then section 8 conducts a sensitivity analysis.

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45 Because of differences like these between $ on FLOP (~“number of chips”) and GWP. the FTM would ideally model bottlenecks separately for both. (This would also allow it to model advanced AI as being disproportionately concentrated on increasing $ on FLOP, rather than assuming they impact all areas of the economy equally. Though FTM does assume advanced AIs are disproportionately concentrated on hardware and software R&D.)
7. Sensitivity analysis

The important takeaways from this section are included in the long summary. I most recommend reading the discussion of trading off training and runtime compute, and perhaps my walk-through of the dynamics in the ‘best guess scenario’.

Thank you to Jaime Sevilla and the team at Epoch for extensive support with this sensitivity analysis.

We performed a few types of sensitivity analysis.

- **Scenario analysis.** We analyse the takeoff trajectories of the Full Takeoff Model (FTM) for conditioning on AGI training requirements being $1e30$ 2020-FLOP vs $1e36$ 2020-FLOP to train AGI vs $1e42$ 2020-FLOP to train AGI. In each case we consider a best-guess, conservative and aggressive takeoff, resulting in nine deterministic scenarios.

- **Timelines analysis.** The prior analyses focus on takeoff speeds, this one focuses on the best-guess implications for AI timelines. Timelines are shorter than Bio Anchors by ~5 - 10 years.
  - Timelines are shorter still, by an additional ~5 years, in a variant of the FTM in which more runtime compute can “substitute” for a lack of training compute, allowing you to automate tasks with smaller training runs.

- **Monte Carlo analysis.** We perform 10,000 simulation runs, each time randomly sampling each parameter. We present summary statistics but do not inspect all the simulations to understand what’s driving the results as in the scenario analysis.

- **Parameter importance analysis.** We varied each parameter from its conservative to its aggressive value, and looked at how this affects takeoff speed.

Assumptions

- Initial growth of AI inputs are chosen to match recent history. E.g. I initially forecast real $\text{S on software R&D will grow at 20\%}, matching its recent growth rate.

- At a certain point, AI readily automates enough economically valuable tasks that “wake up” is triggered.\(^{46}\) Then the best-guess, conservative and aggressive growth rates correspond to those in section 4.

- Best-guess assumptions about bottlenecks from humans and physical capital are as discussed in section 6.

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\(^{46}\) This always happens at 6\% of value-weighted cognitive tasks in goods and services production, except in the Monte Carlo simulation where it can vary from 1\% to 20\%.
Assumptions about the effective FLOP gap match those from section 3, with the constraint that the effective FLOP gap cannot be so large that we predict AI can already readily automate >1% of value-weighted cognitive tasks.47 48

See full list of assumptions and results here.

Scenario analysis

A full list of assumptions and outputs for all scenarios is here.

What’s the takeoff trajectory with my best-guess inputs? This table summarises the answer conditioning on three assumptions about AGI training requirements.

<table>
<thead>
<tr>
<th>FLOP to train AGI using 2020 algorithms</th>
<th>effective FLOP gap50 (OOMs)</th>
<th>Takeoff speed49</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Years from “AI can readily automate 20% of cognitive tasks” to “AI can readily automate 100% of cognitive tasks”</td>
</tr>
<tr>
<td>1e30</td>
<td>2</td>
<td>~3</td>
</tr>
<tr>
<td>1e36</td>
<td>4</td>
<td>~5</td>
</tr>
<tr>
<td>1e42</td>
<td>7</td>
<td>~16</td>
</tr>
</tbody>
</table>

Below I additionally consider “aggressive” and “conservative” scenarios. In aggressive scenarios, all parameters take their aggressive (faster takeoff) values; in conservative scenarios, all parameters take their conservative values. These scenarios feel extreme to me, as many ~independent parameters conspire to make takeoff slow / fast.51 I’d put <1% on the parameters turning out similarly extreme as the conservative scenario, and similarly for the aggressive scenario. Of course there’s model uncertainty in addition to this.

The main takeaways from considering these additional scenarios:

- Takeoff takes ~5X longer in conservative scenarios than in best-guess scenarios mostly due to i) can’t cross the effective FLOP gap by increasing the fraction of FLOP used in the largest training run ii) worse returns to software and hardware R&D.

47 As otherwise AI would be worth more than the ~$50b a year it seems to be currently valued at.
48 The conservative short horizon length scenario violates this constraint, implying that AI can already readily automate 3% of the cognitive tasks. This was to create a larger effective FLOP gap, and so a more gradual takeoff.
49 I list the results for other takeoff metrics here.
50 As above, the startpoint of the effective FLOP gap is “AI can readily perform 20% of cognitive tasks”; the endpoint is “AI can readily perform 100% of cognitive tasks”, i.e. AGI.
51 In particular, the effective FLOP gap, AGI training requirements, R&D returns, R&D investments (which are somewhat related to returns), bottlenecks from physical capital, delays to manufacturing new chip designs, quantity of global FLOP/s produced annually in 2022. OTOH, some of these parameters are significantly more important than others.
- Takeoff happens within a year in all aggressive scenarios as we rapidly cross the effective FLOP gap by increasing the fraction of compute used on the largest training run and automating software R&D.

**1e36 2020-FLOP to train AGI**

I looked at scenarios where AGI training requirements are **1e36 2020-FLOP**, 1 OOM more than median of Bio Anchors medium-horizon anchor for TAI. I set the other parameters to best-guess, conservative, and aggressive values to see the range of takeoff speeds.

<table>
<thead>
<tr>
<th><strong>1e36 2020-FLOP to train AGI</strong></th>
<th>effective FLOP gap</th>
<th><strong>Takeoff metric</strong>&lt;sup&gt;52&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conservative</td>
<td>1e7</td>
<td>27</td>
</tr>
<tr>
<td>Best-guess</td>
<td>1e4</td>
<td>5</td>
</tr>
<tr>
<td>Aggressive</td>
<td>1e2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

<sup>52</sup> I list the results for other takeoff metrics [here](#).
Interesting observations:

- In the best-guess and conservative scenarios, there are multiple years after we’ve ~maxed out the fraction of global FLOP on the largest training run before we’re able to train AGI. During this time AI progress is mostly driven by producing more and better chips globally.
- In the aggressive scenario the majority of AI progress comes from better software (algorithms for training AI).
- In the aggressive scenario we mostly cross the gap by quickly increasing the fraction of chips used in the largest training run.

Walk through of best-guess 1e36 scenario

The best-guess scenario plays out as follows:

- In 2022 the largest training run is ~3e24 2020-FLOP, estimated to cost ~$10m.
• In 2030 we do a ~$3e28 2020-FLOP training run.
  ○ This is ~4 OOMs increase, with ~2.5 OOMs from more $ spend on training and ~1.5 OOMs from software and hardware progress.
• This isn’t sufficient for big economic impact, and spending grows more slowly until 2037 when there’s a $3e30 2020-FLOP training run.
  ○ There’s ~1.5 OOMs from software and hardware progress, and ~0.5 from spending.
  ○ As a result, AIs can readily automate:
    ■ 10% of value-weighted tasks in hardware and software R&D and
    ■ 6% of value-weighted cognitive tasks in the broader economy, which corresponds to >$1tr of value-add. 53
  ○ This triggers “wake up”, when relevant actors realise the economic and strategic potential of AI and invest more quickly.
• In 2039 we do a ~$1e32 2020-FLOP training run.
  ○ This is ~1.5 OOMs increase, with ~1 OOMs from more $ spend on training and ~0.5 OOMs from software and hardware progress.
  ○ AIs have automated ~30% of tasks in hardware and software R&D, and progress is ~1.5X faster as a result. AIs have also automated ~30% of the broader economy, accelerating the growth of the AI chip industry by ~2X.
  ○ These effects of AI automation become more extreme over the following years as AI automates more tasks.
  ○ Shortly after this, the fraction of global FLOP on the largest training run hits its maximum of 10%. 54 Further scale-up of training runs happens via increasing global chip production, improving hardware and improving software.
• In 2044 we do a ~$1e36 2020-FLOP training run that is sufficient to train AGI.
  ○ This is ~1 OOM from more $ spend on training (via scaling up global chip production), ~1 OOM from better hardware and ~2 OOMs from better software.
  ○ Software progress becomes extremely fast as we approach 100% automation of software R&D, such that 1 OOM of software progress happens in the last year before AGI.

See the full list of assumptions (select “Med timelines - Best guess” from the drop down).

1e30 2020-FLOP to train AGI
I looked at scenarios where AGI training requirements are 1e30 2020-FLOP, the low-end of Bio Anchors short-horizon anchor.

The key takeaways are:

53 Human labour receives ~$50tr a year, 3% of this is $1.5tr.
54 More precisely, we’ll do a training run with as many FLOP as you could get by running 10% of all chips non-stop for 1 year.
• In the best-guess scenario AGI is trained in 2028 and takeoff takes 1 - 3 years depending on the metric used. There's still room to increase the fraction of global FLOP on the largest training run when we reach AGI, emphasizing the role of scale up in short timelines scenarios.

• In the conservative scenario AGI is trained around 2034 and takeoff takes 2 - 10 years.

• In the aggressive scenario AGI is trained in 2030 and takeoff takes <1 year.

<table>
<thead>
<tr>
<th>1e30 2020-FLOP to train AGI</th>
<th>effective FLOP gap</th>
<th>Takeoff metric\textsuperscript{55}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conservative</td>
<td>1e4</td>
<td>17 years from “AI could readily automate 20% of cognitive tasks” to “AI could readily automate 100% of cognitive tasks”</td>
</tr>
<tr>
<td>Best-guess</td>
<td>1e2</td>
<td>3 years</td>
</tr>
<tr>
<td>Aggressive</td>
<td>1e1</td>
<td>0.2 years</td>
</tr>
</tbody>
</table>

\textsuperscript{55} I list the results for other takeoff metrics here.
Interesting observations:

- In the best guess scenario we train AGI **0.8 years** before we have enough runtime compute to fully automate software and hardware R&D and **2 years** before we have enough effective compute to fully automate cognitive labour in general.
  - This is because the scenario combines a low training requirement (1e30 2020-FLOP) with an only-moderately low runtime requirement (1e15 2020-FLOP/s).
  - If I use a 1e14 2020-FLOP/s instead, then R&D is fully automated in the same timestep that we get AGI, and we get full automation 1 year later.
  - This illustrates that in scenarios with low AGI training requirements you could be “bottlenecked on runtime compute” such that the explosive improvement in AI capabilities is significantly delayed by not being able to run more AIs.
    - Big caveat: the FTM doesn’t adequately capture the effects of AI improvements after we have AGI, which will **significantly** speed up software progress and reduce the gap between AGI and “AI that could achieve full automation”.
    - Indeed, when I allow **tradeoffs between training and runtime compute**, which implies benefits to training better AIs past full automation, the gap between AGI and “AI that could achieve full automation” reduces to < 1 year.
- In the conservative scenario
  - We quickly max out the fraction of AI chips in a 1e26 training run. After this, better AI must come from more $ spend, better hardware and better software.
  - But severe “parallelisation penalties” mean that it’s hard to speed up the slow progress in software and hardware R&D even by raising spending after wake-up.
  - And people are slow to ramp up spending (perhaps because it’s hard to capture the profits from AI applications).
  - AI automation gradually accelerates progress over a couple of decades to get us to a 1e30 FLOP training run.
- In the aggressive scenario
  - “Wake up” is triggered in 2028:
    - Automating 6% of cognitive tasks requires 5e28 2020-FLOP, only 20X less than AGI.
    - This is a ~4 OOM increase compared to 2022
      - 2 OOMs from software, 0.5 OOMs from hardware, 1 OOMs from $ on FLOP
By the time "wake up" occurs, the largest training run still only uses ~1/10,000 of global FLOP.

We develop AGI later that same year by very rapidly increasing the fraction of FLOP in the largest training run and using AI to do software R&D.

1e42 2020-FLOP to train AGI

I looked at scenarios where AGI training requirements are 1e42 2020-FLOP, 1 OOM more than median of Bio Anchors evolutionary anchor for TAI.

The key takeaways are:

- In the best-guess scenario AGI is trained in 2064 and takeoff takes 6 - 20 years depending on the metric used.
- In the conservative scenario AGI is not trained before 2100.
- In the aggressive scenario AGI is trained in 2056 and takeoff takes 1 - 3 years depending on the metric used.

<table>
<thead>
<tr>
<th>1e42 2020-FLOP to train AGI</th>
<th>effective FLOP gap</th>
<th>Takeoff metric(^{56})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Years from “AI could readily automate 20% of cognitive tasks” to “AI could readily automate 100% of cognitive tasks”</td>
</tr>
<tr>
<td>Conservative</td>
<td>1e10</td>
<td>50</td>
</tr>
<tr>
<td>Best-guess</td>
<td>1e6</td>
<td>16</td>
</tr>
<tr>
<td>Aggressive</td>
<td>1e3</td>
<td>0.9</td>
</tr>
</tbody>
</table>

\(^{56}\) I list the results for other takeoff metrics here.
Walk through of conservative 1e42 scenario

In this scenario:

- "Wake up" is triggered in 2050:
  - Hardware and software progress are about half as quick as in best-guess scenarios due to poor R&D returns; the quality of hardware and software double every 5 - 6 years.
  - Due to a huge effective FLOP gap, wake up is triggered by a training run of \(~1e29\) 2020-FLOP. (This is 14 OOMs below the AGI training requirements.)
  - We achieve this training run mostly by spending more on training.
- Subsequent AI progress is pretty slow for a few reasons:
  - Hardware and software progress continues to be slow, for two reasons.
    - Poor R&D returns.
The fraction of economy doing hardware and software R&D hit their caps (1% GWP). Human R&D investments stop growing after this point, and hardware and software progress become even slower.

- We can no longer increase the fraction of compute on the largest training run.
- Increasing global chip manufacturing capacity is slow.
- Impact of AI automation on hardware R&D progress is bottlenecked by tasks that require physical capital (e.g. experiments).
- As a result we only automate 20% of (2020 value-weighted) cognitive tasks in ~2058 and still haven’t automated all of them by 2100. This doubles the rate of GDP growth, but doesn’t speed up AI progress that much due to bottlenecks.
- AI progress looks mostly like a continuation of standard economic growth trends until a decade of so before AGI, when it starts to accelerate R&D progress.

- The scenario is unrealistic in how much the fraction of GWP spent on AI chips globally grows - ~3 OOMs from its current ~$20b - before we get AGI. This is because the “cap” on the fraction of spending was set exogenously for scenarios where AGI is at most a couple of decades after “wake up”.

See the full list of assumptions (select “Very long timelines - Conservative” from the drop down).

Effect on timelines

The FTM contains a number of dynamics that shorten timelines compared to Bio Anchors.

First I explore how these dynamics play out if AGI training requirements are close to the median implied by Bio Anchors. Then I consider the timelines effect in a wider variety of cases.

The FTM shortens timelines by ~six years for median training requirements

Let’s assume AGI requires 1e36 2020-FLOP to train. This anchors off the Bio Anchors’ medium horizon NN which puts TAI at ~1e35 and is close to the Bio Anchors median. I increase this by 1 OOM because AGI is a higher bar than TAI.

How does the FTM’s best-guess takeoff scenario differ from the best-guess scenario in Bio Anchors?

There are three major differences:

- **Speed-up from AI automation.**
  - **Hardware and software.** Initially the growth of FLOP/$ and 2020-FLOP per FLOP are very similar between Bio Anchors and FTM. But in the FTM AI automation significantly speeds progress.
    - How large is this effect? When I make the R&D runtime requirements impossibly high in the playground, this delays AGI by ~5 years.
- $ on FLOP: AI automation also increases the number of chips produced each year, increasing the $ spent on the largest training run.\textsuperscript{57}
  - How large is this effect?
    - How large is this effect? It increases the largest training run size by $\sim 2X$.\textsuperscript{58}
    - This shortens timelines by $\sim 0.5 \text{ year}$.\textsuperscript{59}
- Faster ramp-up of % GWP spent on a training run.
  - FTM assumes that soon after "wake up" begins we'll use 10% of global FLOP on the largest training run and the real $ spending on global FLOP will double every $\sim 3$ years. This results in a more aggressive forecast than Bio Anchors, if "wake up" begins early enough.
    - How large is this effect? Ignoring the effects of AI automation, this increases the size of the largest training run by $\sim 10X$.\textsuperscript{60}
    - This shortens timelines by $\sim 2 \text{ years}$.

There are a few other minor differences, which overall push towards the FTM having longer timelines.\textsuperscript{61} The differences in hardware, software and $ on the largest training run are shown below:

\textsuperscript{57} In the model this chain of causality is AI automation $\rightarrow$ more GWP $\rightarrow$ more $ investment in producing AI chips.
\textsuperscript{58} Eye-balling the curve in the "money spent on training" graph in the playground, it looks like $\sim 2X$ is spent on training due to AI automation by the time we have AGI.
\textsuperscript{59} I halved the "max fraction of global FLOP on largest training run" in the playground; this delayed timelines by 0.5 years.
\textsuperscript{60} Comparing the "training compute investment ($)" between Bio Anchors and the FTM in the playground.
\textsuperscript{61} FLOP/$ starts slightly higher in FTM; though this is more than cancelled out by a lag before new chip designs are deployed and slower pre-automation rate of hardware progress. The quantitative predictions of Bio Anchors exclude software progress before 2025, whereas FTM excludes it only before 2022; though this is roughly cancelled out by the slower pre-automation rate of software progress in FTM. Bio Anchors caps software progress after $\sim 3$ OOMs (it varies for different anchors), which delays timelines somewhat. Lastly, hardware and software progress slow over time in the FTM as we approach ultimate limits to progress.
All together, the FTM shortens best-guess timelines by about ~six years from ~2050 to ~2044.\(^{62}\)

Of course, the change would be smaller if we had a smaller effective FLOP gap. But in that case takeoff would be even faster. Conversely, a larger effective FLOP gap would make takeoff slower but shorten timelines by even more. It is possible to lengthen timelines and slow down takeoff by changing parameters other than the effective FLOP gap.\(^{63}\)

The FTM shortens timelines compared to Bio Anchors

What about if we don’t want to anchor off the median training requirements estimate from Bio Anchors?

To more thoroughly investigate the shift in best-guess timelines, we did the following:

- Get Bio Anchors predictions for TAI timelines for a variety of different training requirements:
  - Look at the median TAI training requirements for each ‘path’ in Bio Anchors.\(^{64}\)
  - Note down the median year when the TAI is affordable, according to each path.\(^{65}\)
- Get FTM predictions for AGI timelines, anchoring off those same training requirements:
  - Assume AGI requires 1 OOM more training FLOP than TAI.

\(^{62}\) Bio-anchors best-guess parameters put 50% on medium horizon being affordable by 2049; FTM best-guess forecasts AGI will happen in 2041.

\(^{63}\) For example higher training requirements for AGI, worse returns to software and hardware, lower estimates of the world’s ability to grow real inputs to AI.

\(^{64}\) Bottom left graph here.

\(^{65}\) Top left graph here. More specifically, note down the first year when the probability of TAI being affordable exceeds 50%, according to that path.
○ Make an assumption about the best-guess effective FLOP gap, given those training requirements.
○ Run the FTM to see when AGI is trained.

- For each path, compare the Bio Anchors timelines prediction to the FTM timelines prediction.
  ○ AGI being a higher bar also means the shift in timelines is slightly bigger than what the table suggests.

<table>
<thead>
<tr>
<th>Path</th>
<th>FLOP to train TAI using 2020 algorithms</th>
<th>Bio-anchors timelines Year of TAI</th>
<th>Effective FLOP gap</th>
<th>FTM timelines Year when AI could fully automate cognitive labour</th>
<th>Timelines shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime anchor</td>
<td>6e28</td>
<td>2032</td>
<td>1.5</td>
<td>2032 (2028)^66</td>
<td>0 (5)</td>
</tr>
<tr>
<td>Short horizon NN</td>
<td>2e32</td>
<td>2043</td>
<td>3</td>
<td>2037</td>
<td>6</td>
</tr>
<tr>
<td>Genome-like model</td>
<td>6e33</td>
<td>2048</td>
<td>3.5</td>
<td>2041</td>
<td>7</td>
</tr>
<tr>
<td>Medium horizon NN</td>
<td>8e34</td>
<td>2050</td>
<td>4</td>
<td>2044</td>
<td>6</td>
</tr>
<tr>
<td>Long horizon NN</td>
<td>6e37</td>
<td>2062</td>
<td>6</td>
<td>2050</td>
<td>12</td>
</tr>
<tr>
<td>Evolution anchor</td>
<td>2e41</td>
<td>2093</td>
<td>7</td>
<td>2069</td>
<td>24</td>
</tr>
</tbody>
</table>

These timelines shifts are significant: ~5 years for low training requirements and >10 years for high training requirements. This is especially significant given that the forecasting target of the FTM is a significantly higher level of AI: it requires we train AI that can readily perform 100% of cognitive tasks, which is a higher bar than TAI, and that we have enough runtime compute to actually replace all human cognitive labour.

Note that I assumed the effective FLOP gap was bigger in scenarios with higher training requirements, matching my discussion above. This is one reason why the timelines shift is bigger in those scenarios.

^66 I’ve put the year of AGI in brackets, as this is more comparable with the year of TAI and happens many years before full automation due to limited runtime compute. I think the 2033 is too late for full automation, as we used the best-guess 2e16 2020-FLOP/s for AGI runtime requirements in 2020, and the correct answer is probably lower if you can train AGI with 6e28 2020-FLOP.
I believe the shift is especially large for the Evolution anchor because Bio Anchors best-guess assumes that growth in $ on FLOP and software peters out by ~2060, and so the size of training runs grows slowly from 2060-2090. We hit no analogous slowdown in the FTM.

These timelines shifts are very sensitive to the size of the effective FLOP gap and would be very different for other reasonable choices.

- If you made the effective FLOP gap smaller you would get a smaller shift in timelines for two reasons: i) “wake up” would happen later and so AI investments would grow less quickly, ii) significant AI automation would happen later. Also, takeoff would be faster as it would take less time to cross the effective FLOP gap.
- If you made all the effective FLOP gaps bigger you’d get a bigger shift in timelines and takeoff would be slower.

The next section discusses a consideration that shortens timelines even more.

Trading off runtime compute and training compute

Many thanks to Paul Christiano for prompting me to consider this.

What does it mean to trade-off runtime and training compute?

Let’s suppose that training AGI requires 1e36 FLOP and you can then run AGI on 1e16 FLOP/s. If you could only muster a training run of 1e35 FLOP, could you still get an AGI’ output from the resultant system if you used 1e17 FLOP/s at runtime? Perhaps you can make up for the 10X smaller training run by using 10X more runtime compute.

You could have the AI think for longer, or you could run multiple copies. This would allow techniques like “generate multiple solutions to a problem and then pick the best”, or “break the problem into sub-parts and solve each sub-part separately”. Perhaps performance would be the same or better using 1e35 FLOP for training and 1e17 FLOP/s at runtime, compared to using 1e36 FLOP for training and 1e16 FLOP/s at runtime.

Thus far, the FTM has ignored this possibility. If automating the final cognitive task requires 1e36 2020-FLOP but your biggest training run to date is 1e35 2020-FLOP then AI cannot produce any output at that task, no matter how much runtime compute you have to hand. (So says the FTM.) Even if you have huge amounts of runtime compute lying around, you cannot leverage it to reduce your training requirements.

Here’s another way to think about the tradeoff: Before we can cheaply run an AI to perform a given task, we’ll be able to very expensively run an AI to perform the task. At first, the AI will require loads of runtime compute to do the task so it’s expensive; then as AIs improve (which
translates to “training runs get bigger” in this framework’s ontology) we need less runtime compute and it becomes cheaper.

We can trade-off runtime and training compute

There are a few reasons to think that these implications of the FTM thus far are unrealistic.

- **Empirical evidence.** It’s often possible in practice to use additional runtime compute to improve performance, reducing the training FLOP needed to achieve any given level of performance. Some examples:
  - *Monte Carlo Tree Search.* If you search through a deeper tree, that uses more runtime compute and improves performance.
    - [Jones (2021)](https://example.com) varies the depth of tree search of AlphaZero to study its performance at various combinations of runtime and training compute. He finds that using 10X more training compute reduces the runtime compute needed to achieve a level of performance by 15X.
    - This was possible for ~2 OOMs of trade-off before additional runtime hit diminishing returns.
  - *Have multiple attempts at the problem.* If you generate multiple solutions and then submit the best ones, this uses more runtime compute and improves performance.
    - AlphaCode sees very significant improvements through this avenue.
      - It generates multiple possible solutions, some are ruled out via unit tests, and then ten solutions are submitted.
      - In [figure 6](https://example.com), generating one million samples rather than one thousand has a similar effect on performance to increasing the parameter count by 1000X. This means that keeping runtime compute constant, increasing model size didn’t improve performance (in this range)!
      - A similar quantitative result is obtained in OpenAI’s [codex paper](https://example.com).
      - These large benefits of additional runtime compute probably stem from the ability to test generated solutions on unit tests before submission.
    - [WebGPT](https://example.com) found that best-of-n sampling improves results at runtime.68
    - OpenAI found that training a verifier to rank 100 solutions to math problems improved performance as much as increasing model size 30X.
      - That’s 3X more runtime compute making up for ~100X more training FLOP.69
      - But generating more than 400 solutions at test time actually reduced performance.

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67 See figure 6.
68 See section 5.2.
69 I’m assuming here that for GPT scaling a 30X model size increase translates to 100X more training FLOP.
- **Chain of thought prompting.** If language models are encouraged to reason through a problem step by step, this significantly improves performance across a wide range of tasks but also requires more runtime compute.

- **A theoretical argument: No sudden jumps in useful output.**
  - Imagine running the following experiment. First, you do a 1e30 FLOP training run, then try to get as much useful output at some task as possible from your system using (e.g.) 1e31 runtime FLOP. Then you do a slightly bigger training run, say 3e30 FLOP, and again get as much useful output as you can by running it using 1e31 FLOP. You continue to slowly increase your training FLOP, leaving your training FLOP constant.
  - How will the AI’s output at the task change over time? I expect it to improve smoothly. Performance on most benchmarks does this, especially when you’re explicitly optimising for the benchmark during training (e.g. fine-tuning on that specific task).
  - The FTM currently has a contrary implication. It implies that, when the training run is below the threshold for performing the task, you will produce 0 output. When it rises above that threshold, you will suddenly produce a task output that is proportional to your runtime FLOP (and so potentially extremely large).
  - This implication can be removed from the FTM by allowing it to trade-off runtime and training FLOP. Then if the training run is 10X smaller than the threshold, AI can still perform the task but it uses runtime compute (e.g.) 10X less efficiently. That way you’ll initially have small output at the task, via having a small training run that forces you to use your runtime compute very inefficiently.

- **A theoretical model.**
  - David Schneider-Joseph has made a model of this tradeoff in a setting where the task is to correctly predict the next 5000 tokens.
  - In the model, the trade-off can reduce training compute by up to ~3 OOMs, at the cost of using 8 OOMs additional runtime compute.

The empirical evidence typically suggests you should be able to do the tradeoff for at least a few OOMs, but no more without additional tricks; the theoretical model backs this up. The theoretical argument seemingly supports being able to do the tradeoff ~indefinitely.

**We can model this trade-off in the FTM**

Currently here’s how the FTM works:
- Each task t has a training requirement T and a runtime requirement R.
- If the largest training run exceeds T FLOP, you can use R FLOP/s to run one human-equivalent worker doing t. If the largest training run is below T, AI can’t perform t.

Here’s how we can adjust the FTM to incorporate a trade-off between training and runtime compute:
- [Unchanged] Each task t has a training requirement T and a runtime requirement R.
If the largest training run equals \( T' \) FLOP, you can use \( R' \) FLOP/s to run one human-equivalent worker doing \( t \). \( R' = R \ast (T / T')^N \).

- If \( T' = T \), it takes \( R \) FLOP/s to run a human equivalent, as before.
- If \( T' < T \), it takes more than \( R \) FLOP/s to run a human equivalent.
- If \( T' > T \), it takes less than \( R \) FLOP/s to run a human equivalent.
- The bigger your training run, the more efficiently you can perform the task.
- \( n \) controls the quantitative tradeoff between runtime and training FLOP.
  - If \( N = 1 \), a 10X smaller training run increases runtime compute by 10X.
  - If \( N = 2 \), a 10X smaller training run increases runtime compute by 100X.
  - If \( N = 3 \), a 10X smaller training run increases runtime compute by 1000X.
  - What is \( N \)?
    - The above empirical ML evidence can be interpreted to give estimates of \( N \).
    - I estimate \( N \) using some of this ML evidence, and some other types of evidence, here.

Trading-off training and runtime compute generically shortens timelines

Without this trade-off, fully automating (e.g.) software R&D requires two conditions:
- Enough training FLOP for AI to perform all tasks.
- Enough runtime FLOP to run more AIs than humans at each task.

If you fail either condition, you can't fully automate software R&D. See diagram.
With the trade-off, you can fall short of one condition (e.g. not enough training FLOP) but make up for it with the other (e.g. more than enough runtime FLOP). See diagram.

It is strictly easier to fully automate software R&D with the tradeoff. The shaded region with tradeoffs is a superset of the shaded region without tradeoffs.

In a similar way, if it’s possible to tradeoff training and runtime compute then lower levels of automation will be strictly easier to achieve and they will occur earlier in time. So, quite generically, having a tradeoff shortens timelines.

Trading-off training and runtime compute can significantly shorten timelines if you have ~large training requirements

Let's say AGI requires 1e36 FLOP, one OOM more than the Bio Anchors median for TAI. And let's say it runs in 1e16 FLOP/s. In this case, I think the possibility of trading-off runtime and training compute would significantly shorten timelines.

Let's assume that 10% of a year's FLOP are ultimately used to train AGI. In that year, 1e37 FLOP were available in total. Let's also assume that 10% of those FLOP are used to run AGIs
doing AI software R&D: 1e36 FLOP.\textsuperscript{70} You could run ~3e12 AGIs doing software R&D (and more in total).\textsuperscript{71}

3 trillion human-equivalents would be much more than sufficient to fully automate software R&D. I would guess there are 10,000 people pushing forward SOTA AI (10X DeepMind’s team). That’s 300 million times as many AGIs as the current workforce!\textsuperscript{72} Let’s assume a 100X expansion in the software R&D workforce by the time of AGI, reducing the discrepancy to three million.

But this underestimates the discrepancy. AGIs will have several significant productivity advantages over humans: thinking faster, better motivations, less leisure and sleep, you can copy the most productive workers. Let’s say this all boosts their productivity by 30X, taking the discrepancy to 100 million. AGIs would be 100 million times as productive at software R&D as the human workforce.

So by the time we train AGI, we’ll have much more runtime compute than we need to fully automate software R&D. If tradeoffs are allowed, we could leverage that to fully automate software R&D with a smaller training run. How much smaller?

Let’s suppose that at some earlier time we had 1000X less compute. So our biggest training run is 1000X smaller at 1e33 FLOP, 1000X below the AGI training requirement. We also have 1000X less runtime compute: we used to have 100 million times more runtime compute than we needed, now we have 100,000X as much. So our training compute is 1000X below the requirement, but our runtime compute is still 100,000X above the requirement of matching human output.

If we can trade-off 1 OOM of training for 1 OOM of runtime, we’ll be able to run AIs whose software R&D output is 100X that of humans. This means we could fully automate software R&D with a training run 3 OOMs smaller than without tradeoffs (1e33 rather than 1e36).

This is just an example, you can play around with different numbers in this sheet. It suggests though that if AGI has large training requirements, we will be ~swimming in runtime FLOP for software R&D long before we’re able to train AGI. If in addition we can trade-off runtime and training FLOP, we’ll be able to ~fully automate AI software R&D by running loads and loads of pre-AGI systems. This would significantly shorten timelines by reducing the largest training run we have to run by several OOMs.

\textsuperscript{70} Why 10%? After “wake up”, I expect people to be keen to increase software R&D investment, and AIs are much easier to ‘redirect’ towards this purpose than humans.

\textsuperscript{71} FLOP available in a year / AGI FLOP per year = FLOP available in a year / (AGI FLOP/s * seconds in a year) = 1e36 / (1e16 * ~3e7) = ~3e12.

\textsuperscript{72} 3e12/1e4 = 3e8
The FTM implies that trading-off training and runtime compute significantly shortens timelines

The above dynamic plays out in the FTM. For each task, rather than needing to exceed a certain training requirement, you can instead automate it with a smaller training run by using more runtime FLOP. The logic of the above section is replicated for each task. Allowing trade-offs brings forward in time the whole pattern of automation.

Without tradeoffs, AGI is first trained in ~2040 with my best guess parameters. With tradeoffs, it’s trained much sooner. The shift depends on \( N \), how many extra OOMs of runtime compute you need to make up for having 1 OOM less training compute.

<table>
<thead>
<tr>
<th>( N )</th>
<th>Year when AI could readily automate 100% of cognitive tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>2038</td>
</tr>
<tr>
<td>2</td>
<td>2040</td>
</tr>
<tr>
<td>6</td>
<td>2042</td>
</tr>
<tr>
<td>No tradeoff</td>
<td>2044</td>
</tr>
</tbody>
</table>

The swing here is significant and this merits further investigation. The basic story, taking the \( N=0.5 \) case as an example, is that there aren’t that many AI software R&D workers and so we’re able to achieve full R&D automation in 2037 – seven years earlier than with no tradeoff! – by doing a 3e33 training run (2.5 OOMs less 2020-FLOP than the AGI training requirements) and using 2.5*0.5=1.25 OOMs more runtime compute than would be needed to fully automate R&D after a 1e36 training run. Then fast software progress means that AI could readily automate 100% of cognitive tasks 1 year later.

Another effect of this is to smooth software progress as we approach full automation. Without a tradeoff, output on the final cognitive task can discontinuously increase by multiple OOMs when it is first automated; the tradeoff smooths this transition as when a task is first automated it is normally via a tradeoff and so there is limited runtime compute for the task. Compare the \( N=0.5 \) scenario with the “no tradeoff” scenario:

---

\(^{73}\) I’ve been referring to the training and runtime requirements of the final task as the AGI requirements. Other tasks have lower training and runtime requirements than this.
Top: tradeoff with $N=0.5$. Bottom: no tradeoff. (All other parameters take best-guess values.) Notice the sudden increase in the level of software (green line) when software R&D is fully automated in the bottom scenario. This sudden jump is smoothed out in the top scenario by the tradeoff.
Capping the extent to the tradeoff reduces its effect

There are some potential reasons to doubt the above results:

1. Software progress counts for ‘double’ when you have a trade-off.
   a. If you improve software by 10X in the FTM, that increases both training and runtime efficiency by 10X.
   b. My starting value for AGI runtime requirements is 1e17 FLOP/s. Based on one-time gains, I reduce this to ~1e16 FLOP/s for goods and services and ~5e15 FLOP/s for R&D. With ~3 OOMs of software progress before AGI, these reduce to ~1e13 FLOP/s. So the FTM is expecting AGI runtime requirements to be fairly low by the time AGI is near. The plentiful runtime compute is then traded-off to reduce training requirements.
   c. If you don’t think AGI runtime requirements will become this low, or you think there’s something suspicious about trading-off runtime efficiency improvements to reduce your training requirements, you should be sceptical of the above.

2. Trading off between training and runtime might only be possible for a few OOMs.
   a. If you’re 2 OOMs short of the training requirement, you can make up for it with more runtime compute, but if you’re 6 OOMs short there’s nothing you can do.
   b. Indeed, in all the empirical examples above, and in the theoretical model, the ability to improve performance using more runtime FLOP peters out after a few OOMs.
      i. AlphaCode is a possible exception: the benefits increase over at least 6 OOMs. This is probably because tested generated solutions to see if they’re good before submitting. The lesson might be: you can tradeoff for more OOMs when you can easily evaluate output.
      ii. This isn’t conclusive: perhaps if we try we’ll find new techniques that allow the trade-off to continue for longer.
   c. In these simulations, the trade-off operates over many OOMs. In the top N=0.5 scenario full automation is achieved with 7 OOMs fewer training 2020-FLOP than the training requirement; in the middle scenario it’s 3 OOMs fewer.
   d. How far should the trade-off continue for before it’s capped?
      i. The above examples suggest 2-3 OOMs is pretty common.
      ii. But some of the techniques used are things humans already do to enhance their output. E.g. “chain of thought” and “evaluate and revise their own work”. We should only include a trade-off if it will allow AIs to substitute “more thinking time” for “being smarter” beyond what humans already do.
      iii. Some existing techniques do go beyond what humans typically do, e.g. best-of-N is normally too expensive to be practical for humans; but might well be so for AIs. In addition, I expect researchers to develop novel techniques for leveraging cheap runtime compute to improve performance. We should include these techniques in the trade-off.
      iv. Overall, I’m currently inclined to see a cap after ~1-2 OOMs as a best-guess, with 3 OOMs as aggressive and 0 OOMs as conservative.
   e. This is the reason I find the most convincing for distrusting these results.
3. The FTM simulation overestimates the runtime FLOP available for AI early on.
   a. The simulation assumes all FLOP in the world is initially used for running AIs.
   b. This isn’t true, I believe the true number is closer to 1-10%.
   c. In addition, the FTM best-guess assumptions are that after “wake up” the fraction of global FLOP used to run AIs doing software R&D rises quickly to a cap of 20%, which is a lot.
   d. However I don’t think this factor is significant.
      i. We reran the three simulations with 100X less runtime FLOP and timelines were delayed by 1-2 years in each case. So there was still a large shortening in timelines from the tradeoff, even using conservative amounts of runtime FLOP.
      ii. The FTM significantly overestimates the number of human workers in software and hardware R&D in 2022 (1.6 million and 160 million), which pushes in the opposite direction.

To account for these worries, especially number 2, we also ran simulations where the trade-off could only happen for 1.5 OOMs in either direction.

<table>
<thead>
<tr>
<th>AGI takes $1e36$ FLOP to train with 2020 algorithms</th>
<th>Year when AI could readily automate 100% of cognitive tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>No cap on tradeoff</td>
</tr>
<tr>
<td>0.5</td>
<td>2038</td>
</tr>
<tr>
<td>2</td>
<td>2040</td>
</tr>
<tr>
<td>6</td>
<td>2042</td>
</tr>
<tr>
<td>No tradeoff</td>
<td>2044</td>
</tr>
</tbody>
</table>

Repeating the above analysis for a short timelines scenario
The above numbers all assumed AGI training requirements = $1e36$ 2020-FLOP, runtime requirements = $2e16$ 2020-FLOP/s.

Let’s redo the analysis for training requirements = $1e30$ 2020-FLOP, runtime requirements = $1e15$ 2020-FLOP/s.
<table>
<thead>
<tr>
<th>AGI takes $1\times10^{30}$ FLOP to train with 2020 algorithms</th>
<th>Year when AI could readily automate 100% of cognitive tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>No cap on tradeoff</td>
</tr>
<tr>
<td>0.5</td>
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<td>2</td>
<td>2030</td>
</tr>
<tr>
<td>6</td>
<td>2030</td>
</tr>
<tr>
<td>No tradeoff</td>
<td>2032</td>
</tr>
</tbody>
</table>

There’s a smaller effect, because runtime compute is more of a bottleneck when training requirements are low. So we can’t trade off as much runtime compute to get full automation sooner.

Am I including the tradeoff in the results?

There’s no tradeoff in the scenario analysis above, but it is included below in the Monte Carlo and the parameter importance analysis.

Monte Carlo analysis

We ran 1000 simulations, each time randomly sampling each parameter between its “conservative” and “aggressive” values (listed here).\(^{74}\) The only exception is AGI training requirements, which are sampled from the Bio Anchors distribution.

We encoded correlations between the sampled values of each parameter. The main correlations are:\(^{75}\)

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\(^{74}\) The sampling distribution is a mixture of two distributions. It places 50% weight on a log-uniform distribution between the parameter’s “conservative” and “best-guess” value, and 50% weight on a log-uniform distribution between its “best-guess” and “aggressive” values.

\(^{75}\) These high-level correlations, and others, are recorded here; the full matrix of correlation is here (see “Rank correlations: click here to view”). I think I’ve overestimated the correlations between these inputs, extremizing the tail outcomes. On the other hand, my sampling procedure means the parameter values never fall outside the “conservative” to “aggressive” range, which pushes in the opposite direction.
- **Strong correlation between growth of AI investments.** If spending in AI software R&D grows quickly, I also expect spending on hardware R&D and AI chips to grow more quickly.

- **Strong correlation between AGI training requirements with 2020 algorithms and total possible software progress before hitting physical limits.** Human lifetime learning gives a theoretical limit on how efficient learning can be; ~1e24 FLOP. If AGI would currently take 1e30 FLOP to train, that suggests 6 OOMs of improvement left; if it would take 1e40, that suggests 16 OOMs left.

- **Strong correlation between the AGI training requirements and the cap on trading of training and runtime compute.** I expect the ability to do these tradeoffs to increase significantly over time as we develop new techniques for it, and this correlation is a hacky way to incorporate that.

- **Medium correlation between AGI training requirements and the effective FLOP gap.** If AGI requires “long horizon” training, or some other high-cost approach to training, that increases my probability that 20% of tasks will be automated with much less effective training compute than AGI.

- **Medium correlation between AGI training requirements and how much easier it is to train AI that can perform all R&D tasks than to train AGI.** If AGI requires “long horizon” training, or some other high-cost approach to training, that increases my probability that you can perform all hardware and software R&D tasks with less training FLOP.

- **Medium correlation between AGI training requirements and growth of AI investments.** If AGI training requirements are lower, it should be easier to grow AI investments quickly as they’re starting from a lower base.
  - This also captures that fact that lower training requirements → smaller effective FLOP gap → AI capabilities increase more with each increment of investment → more incentive to quickly grow AI investments.\(^{76}\)

- **Medium correlation between different ‘bottlenecking’ parameters.** If non-automated human tasks or physical capital are a significant bottleneck in GDP they’re more likely to be so in R&D as well.

- **Weak correlation between different R&D returns.**
  - Good returns to hardware R&D → cheaper to run big ML experiments which are useful for software R&D → better returns to software R&D.
  - Good returns to hardware R&D → techno optimists were right about hardware progress → they’re relatively more likely to be right about software progress. (I’m assuming here that people’s views on the likely returns from different areas of AI R&D are correlated.)

- **Weak correlation between growth of AI investments and R&D returns.** If R&D returns are higher, the per-$ payoff from AI investments is higher.

- **Weak correlation between growth of AI investments and the absence of bottlenecks.** If there aren’t significant bottlenecks, the per-$ payoff from AI investments is higher.

\(^{76}\) We could have represented this via correlation between the effective FLOP gap and AI investments, but didn’t for a complicated technical reason.
We resample the parameters until we avoid the implication that AI can already readily automate >1% of the economy or >5% of R&D. This raises the training requirements, while lowering the FLOP gap and cap on the tradeoff between training and runtime compute.

Here are the results, sampling AGI training requirements from the [Bio Anchors best-guess distribution](#) (median ~1e36 FLOP to train AGI with 2020 algorithms).\(^77\)

<table>
<thead>
<tr>
<th>Percentile</th>
<th>AI timelines</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First year when AI can readily automate 100% of cognitive tasks in the general economy.</td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>2025.7</td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>2029.6</td>
<td></td>
</tr>
<tr>
<td>20%</td>
<td>2032.7</td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>2043.3</td>
<td></td>
</tr>
<tr>
<td>80%</td>
<td>2070.3</td>
<td></td>
</tr>
<tr>
<td>90%</td>
<td>≥ 2100</td>
<td></td>
</tr>
<tr>
<td>99%</td>
<td>≥ 2100</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Percentile(^78)</th>
<th>Takeoff speed</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tasks in the general economy.</td>
<td>Tasks in software and hardware R&amp;D.</td>
</tr>
<tr>
<td>1%</td>
<td>0.3</td>
<td>0.9</td>
</tr>
<tr>
<td>10%</td>
<td>0.8</td>
<td>1.6</td>
</tr>
<tr>
<td>20%</td>
<td>1.2</td>
<td>2.2</td>
</tr>
<tr>
<td>50%</td>
<td>2.9</td>
<td>4.3</td>
</tr>
<tr>
<td>80%</td>
<td>7.6</td>
<td>9.6</td>
</tr>
<tr>
<td>90%</td>
<td>12.5</td>
<td>14.6</td>
</tr>
</tbody>
</table>

\(^77\) We assume AGI training requirements are 1 OOM higher than TAI training requirements, and reduce the probability of “you need more compute than evolution” from 10% to 4%.

\(^78\) This is all conditional on AGI before 2100.
The median AGI year is later than in the best-guess scenario (2043 vs 2040), and the takeoff is faster than in the best-guess scenario. I believe the key reason is that the median effective FLOP gap in the Monte Carlo is smaller than my best-guess effective FLOP gap (~3.5 OOMs vs 4 OOMs). This makes takeoff faster and lengthens AI timelines.

Why is the median effective FLOP gap smaller than the best-guess effective FLOP gap? We sampled the gap from a distribution in which the median was 4 OOMs, but we resampled when the effective FLOP gap was so large that it implied that AI can already readily automate either >1% of the economy or >5% of R&D. This meant we resampled large effective FLOP gaps more commonly than small effective FLOP gaps, skewing the median of the resultant distribution down.

Why think that it is this smaller median effective FLOP gap that explains the difference between the median results of the Monte Carlo and the best guess? Because when we change the effective FLOP gap in the best guess scenario to match the Monte Carlo median, the discrepancy disappears. In other words, when you run one deterministic simulation where each parameter takes its median value from the Monte Carlo, the results are similar to the median results from the entire Monte Carlo.

<table>
<thead>
<tr>
<th>AGI year</th>
<th>Takeoff speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic sim: all params take median values from Monte Carlo</td>
<td>2043.6</td>
</tr>
<tr>
<td>Monte Carlo median</td>
<td>2042.8</td>
</tr>
</tbody>
</table>

Those with shorter timelines may be more interested in the Monte Carlo results when sampling AGI training requirements from an alternative distribution with a more aggressive distribution that has a median of ~1e31.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>AI timelines</th>
</tr>
</thead>
<tbody>
<tr>
<td>First year when AI can readily automate 100% of cognitive tasks in the general economy.</td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>2024.8</td>
</tr>
<tr>
<td>10%</td>
<td>2027</td>
</tr>
</tbody>
</table>

79 For all other parameters, the median Monte Carlo value is very similar to the best-guess value.
80 Years from “AI can readily perform 10% of cognitive tasks” to “you can run 10 billion AGIs”
<table>
<thead>
<tr>
<th>Percentile</th>
<th>20%</th>
<th>50%</th>
<th>80%</th>
<th>90%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2028.6</td>
<td>2033.7</td>
<td>2044.1</td>
<td>2054.9</td>
<td>≥ 2100</td>
</tr>
</tbody>
</table>

### Takeoff speed

<table>
<thead>
<tr>
<th>Percentile</th>
<th>1%</th>
<th>10%</th>
<th>20%</th>
<th>50%</th>
<th>80%</th>
<th>90%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tasks in the general economy.</td>
<td>0.2</td>
<td>0.5</td>
<td>0.7</td>
<td>1.7</td>
<td>3.9</td>
<td>6.1</td>
<td>20</td>
</tr>
<tr>
<td>Tasks in software and hardware R&amp;D.</td>
<td>1.1</td>
<td>1.7</td>
<td>2.2</td>
<td>3.7</td>
<td>6.4</td>
<td>9.1</td>
<td>24.2</td>
</tr>
</tbody>
</table>

Will there be a 4-year GWP doubling before the first 1-year GWP doubling? Each simulation run calculates the Gross World Product (GWP) in each timestep, which means we can calculate the fraction of runs in which a 4-year GWP doubling finishes before the start of the first 1-year GWP doubling. This happens in 77% of runs with the Bio Anchors best-guess distribution and 68% of runs with the Bio Anchors aggressive distribution.

My personally probability for this is somewhat lower than these numbers because I expect deployment delays to be longer for pre-AGI systems than for superintelligent AGIs.

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81 This is all conditional on AGI before 2100.
Parameter importance analysis

This framework is complicated. There are many parameters. To test their importance, we varied each from a conservative to an aggressive value and looked at the resultant change in various takeoff speed metrics. All other parameters were held fixed at their best-guess values.

Important parameters tend to affect either i) the ability to cross the effective FLOP gap via increasing the fraction of global FLOP on the largest training run or ii) the returns to software and hardware R&D as AI automation increases R&D inputs.

The following table lists parameters from most to least important for takeoff speed. “years from AI 82

<table>
<thead>
<tr>
<th>Parameter</th>
<th>When I vary the parameter, how much does takeoff speed change?</th>
<th>Explanation for parameter importance</th>
</tr>
</thead>
</table>
| AGI training requirements (2020-FLOP)  | Takeoff speed = Years from “AI can readily automate 20% of cognitive tasks” to “AI can readily automate 100% of cognitive tasks”. | - When AGI training FLOP is high, we reach the maximum fraction of global FLOP on a training run before the takeoff startpoint. So increasing the fraction doesn’t contribute to crossing the effective FLOP gap.  
- FTM assumes that hardware and software returns become worse over time. More.  
- Human inputs to software R&D reach their maximum (1% GWP) before AGI, slowing software progress.  
- The quantitative analysis actually ranked this a second most important, but I think that’s a mistake.  
i) It ignored the fact that small AGI training requirements imply a narrow effective FLOP gap.  
ii) It varied training requirements within a narrow range83, underestimating its importance. |
| effective FLOP                         | - This is the ‘distance we need to cover’ during takeoff. If | |

82 The ordering depends on the average difference the parameter makes to several different takeoff speed metrics. These are all AI capability metrics, rather than impact metrics (e.g. GWP impacts). They are the ones listed here before the ‘combined’ column which gives their average.

83 1e33 to 1e40. Why not use a wider range? The analysis uses an effective FLOP gap for 5 OOMs, and this combined with 1e30 AGI training requirements implied that >1% of goods tasks were already automated in 2022.
|gap| it’s twice as far, takeoff takes ~twice as long.\(^{84}\)
| | - It’s **highly uncertain**.

|R&D parallelisation penalty| I.e. If I double research effort, how much faster does R&D progress?
| | - I think this could range from 1.2X to 2X; I’m not aware of good data pinning it down.
| | - This affects how much software and hardware progress accelerates when research efforts rise due to higher human investment and AI automation.

|Software returns| - Highly uncertain
| | - Software updates can be immediately applied to all hardware without delay, making this especially important as progress accelerates.

|R&D vs general economy, automation training requirements| I.e. How much less effective training compute is needed to fully automate the cognitive tasks in AI R&D compared to the general economy?
| | If the difference is significant, AI may be dramatically accelerating AI progress by the time AI can readily automate 20% of cognitive tasks in the general economy.

|Substitutability between different cognitive R&D tasks| - Conservative value means that human bottlenecks significantly limit the impact of partial AI automation
| | - Affects both hardware and software R&D

|Maximum tradeoff between training and runtime compute| A bigger tradeoff the training requirements for AI to fully automate AI R&D, reducing AI timelines and thereby accelerating up takeoff.

|Effective FLOP gap in runtime requirements| I.e. how much more compute do you need to run AI that can perform 100% of cognitive tasks, compared to AI that can perform 20%.
| | This matters in worlds where automation is bottlenecked by a lack of runtime compute; i.e. worlds where AGI training requirements are low.

|g(fraction of GWP spent on compute)| I.e. how quickly can we scale up production of AI chips?
| | This translates ~directly into more FLOP for training AI.

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\(^{84}\) This is a rough rule of thumb, but could be false. Increasing the fraction of global FLOP on a training run might allow us to cross a small effective FLOP gap very quickly; but then if the effective FLOP gap were twice as big we’d max out that fraction and take more than twice as long to cross.
- Current input growth is only ~5%, so AI automation can significantly increase this growth and accelerate progress.

I.e., how long after inventing a new hardware design can we start printing it?

This delays the feedback loop: better AI → more hardware R&D → better AI chips → better AI.

The analysis underestimates the importance of AGI training requirements for the reasons stated in the table. So overall I believe that AGI training requirements is the most important parameter for takeoff speed.

See full results.

8. Limitations of the Full Takeoff Model, and how they affect takeoff speed

This gives more detail on many of the FTM’s limitations and could be worth reading if you want to understand them better.

The Full Takeoff Model (FTM) is limited in many ways. This section looks at its various limitations and asks “does this limitation bias the model towards thinking takeoff is too fast or too slow?”

The following table summarises the limitations and their implications for takeoff speed.

<table>
<thead>
<tr>
<th>Limitation</th>
<th>Implications for takeoff speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deployment delays (that don’t delay future AI capabilities progress)</td>
<td>Makes impact takeoff slower in general; but could make it faster if superhuman AI quickly removes barriers to deployment; creates a scary capability imbalance between “SOTA AI” and “AI that’s integrated in the economy”.</td>
</tr>
<tr>
<td>Deployment delays (that do delay future AI capabilities progress)</td>
<td>Takeoff will be somewhat slower than the predictions of the FTM (months, maybe years)</td>
</tr>
<tr>
<td>Additional bottlenecks to developing AI systems</td>
<td>Could make takeoff faster or slower; overall I think this makes very fast takeoff scenarios less likely.</td>
</tr>
<tr>
<td>There might be a discontinuous jump in AI capabilities</td>
<td>Raises the probability of extremely fast takeoff (e.g. in days, weeks or months).</td>
</tr>
</tbody>
</table>
Focuses on the *transition to AGI* rather than the *aftermath of AGI* | Unclear.
---|---
The FTM implies GWP growth is certain to accelerate with AGI, which many economists disagree with. | Makes fast takeoff less likely.
The FTM assumes that investment in AI R&D vs other areas grows exogenously, rather than modelling actors’ incentives. | Adds uncertainty
Elides the distinction between capabilities takeoff and impact takeoff

Overall, considering these factors:

- Increases my credence in very fast takeoff and very slow takeoff.
- Updates me towards slower capability takeoff overall.
- Updates me towards much less time from “AI has a major economic impact” to “AI poses x-risk”. (This metric has an impact-based startpoint and a capability-based endpoint.)
- Updates me towards longer AI timelines.

There might be delays between developing advanced AI and deploying it

This limitation concerns AI automation. The FTM assumes that, for any trained AI, we deploy it in the economy (/ hardware R&D / software R&D) as soon as we have enough runtime compute to do so.\(^\text{85}\)

But in fact there might be delays to deploying AI systems. Example delays include regulations, concerns about the safety of AI systems, political resistance from workers who would lose their jobs\(^\text{86}\), people distrusting AI, time needed to integrate AI into work flows\(^\text{87}\), and other delays from domestic or international politics.

Some deployment delays will delay future AI progress very directly. For example, delays to deploying AIs that do software R&D will delay future algorithmic progress. And similarly for AIs that would increase FLOP/$ or increase fab throughput (expanding the output of the fab industry and so growing $ on FLOP globally). Other deployment delays won’t cause future AI progress to

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\(^{85}\) I explained this assumption in more detail in section 5.

\(^{86}\) Though this will be less of a problem to the extent that AI automates parts of existing jobs, allowing practitioners to spend more time on other aspects of the job.

\(^{87}\) This last delay I’ve tried to sidestep by stipulating that when I say “AI can perform” a task I mean that AI can *readily* perform that task. If it would take a lot of time to integrate AI into work flows so it can perform a task, then AI can’t readily perform the task. This raises the bar for training “AI that can perform x% of tasks”. 

be delayed. For example, delays in deploying AI to improve healthcare need not slow down subsequent AI progress (though it may if it reduces future AI investments).

To simplify the analysis, let's first consider deployment delays that don't cause future AI progress to be delayed. Then we'll consider deployment delays that do delay future AI progress.

Deployment delays that don't delay future AI progress

Delays like these don't affect capability takeoff speed but may affect impact takeoff speed.\(^{88}\)

There are a few different forms these delays could take, which affect impact takeoff speed in different ways. I'll briefly mention four possibilities.

**Constant Delays.** If there's a constant time delay to deploying systems (relative to the predictions of FTM), then impact takeoff speed doesn't change. All impacts are moved back in time by the same amount.\(^{89}\) A concrete example would be "it takes the government 6 months to approve each new AI for deployment".

**Diminishing Delays.** If there are shorter delays to deploying more capable AIs (e.g. because they can increasingly manage their own rollout), then impact takeoff is faster. The impacts of earlier AIs are delayed by more than the impacts of later AIs, so there's less calendar time from "small impact" to "large impact". An extreme example of this is "pre-AGI systems aren't deployed anywhere in the economy due to regulations; then one year superintelligent AGI takes over and starts colonizing the stars".\(^{90}\)

This fits with the long-run trend of deployment delays shortening over time:

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\(^{88}\) I explain the distinction between these notions of takeoff speed [here](#); I think hard takeoff in *either* sense would be strategically important.

\(^{89}\) If your preferred takeoff speeds metric was a mixture of capabilities and impact metrics, then Constant Delays could affect takeoff speed. For example, the "time from a crazy impressive demo to 20% GWP growth" would increase with Constant Delays (the growth but not the demo would be delayed). But the "time from 5% GWP growth to first training AGI" would decrease with Constant Delays.

\(^{90}\) Another extreme example is the "sonic boom" argument, that falling deployment delays will cause a discontinuous jump in the capability of deployed AIs. Search "sonic boom".
Source.

Source.
This provides evidence for Diminishing Delays and suggests delays for advanced AI could be anywhere from a few months (for some software), to ~10 years (like the internet).

**Growing Delays.** If there are longer delays to deploying more capable AIs (e.g. because it’s harder to check that we trust more capable AIs) then impact takeoff is slower.

**Impact Distributing Delays.** In the previous two possibilities, each AI has the same impact as the FTM predicts but only after some time delay. A third possibility is that each AI’s impact is spread out over many years or decades. A concrete example would be “regulations always mean that each new AI can initially only operate in 10% of industries and it takes 20 years before it can operate in >90%”. This would prevent a fast impact takeoff, no matter how fast capabilities takeoff is.

Notice that if Impact Distributing Delays applied to pre-AGI systems, but stopped applying after AI exceeds a certain capability threshold, then this would again make impact takeoff faster (it’s just like Diminishing Delays). To guarantee slow takeoff, Impact Distributing Delays must apply for AI of any capability level.

Which of these is most plausible?

- Impact Distributing Delays seem plausible, at least when AI intelligence is below human-level.
  - This is closest to what has occurred historically for most tech. Even if the tech is developed very quickly, adoption takes decades.
  - Humans will be (rightly!) cautious to deploy advanced AI in important and power-granting roles in the economy. This could cause many years of delay to rolling out even very capable AI.

- Competition could prevent Impact Distributing Delays from occurring, especially in strategically important areas.
○ If two countries are a few months or years apart in AI capabilities, and superhuman AI grants large amounts of power, then delaying deployment by months or years would risk falling significantly behind.
○ So people may think that they should quickly deploy AI in strategically important areas like R&D, intelligence gathering, and military strategy.
○ If even one region deploys superhuman AIs without any human restrictions then, absent bottlenecks from raw materials, I believe that their technological capabilities, military power, and physical infrastructure would soon be growing extremely quickly.
○ We should coordinate to avoid fast deployments of this kind.
  ● I’m unclear whether Impact Distributing Delays will happen once AI intelligence exceeds that of humans.
    ○ Firstly, there’s the risk of AI takeover.
      ■ If AL takes over, the “humans are cautious” source of Impact Distributing Delays will be removed.
      ■ If the AI makes a sudden successful takeover attempt, then its impact on the world would increase very suddenly. It would be a case of Diminishing Delays.
    ○ Secondly, there’s the possibility that aligned AI becomes capable enough to overcome a previous barrier to deployment.
      ■ Perhaps organizations are biased against incorporating new technologies to their workflows, but AI advisors improve their decision making.
      ■ Perhaps people were scared of deploying advanced AI, but superhuman AI shows them compelling evidence that there is no downside risk.
      ■ Perhaps deployment was previously blocked by incumbent workers and capital owners worrying about losing their relative position in society. But then aligned superhuman AI proposes a deployment plan that is in everyone’s interests and pitches it extremely convincingly.
      ■ Perhaps deployment was previously bottlenecked by human bureaucratic processes, but aligned superhuman AI invents technology for uploading humans to computers so they can think 1000X faster when working.
      ■ Again, these would be examples of Diminishing Delays.
      ■ Importantly, the effective FLOP gap was defined to hold constant the necessary deployment delay.
        ● The definition of “AI that could readily automate x% of tasks” bakes in that it would be profitable to automate each task with <1 year of engineering and rearranging work flows.
        ● But actual deployment delays may get closer to necessary deployment delays as AI advances.
  ● Growing Delays seems possible if we become increasingly cautious about deploying increasingly advanced AI. Though, as discussed above, AI takeover or competitive dynamics might curtail this once AI is above human-level intelligence.

My bottom line here is these delays:
● Will probably slow down impact takeoff speed, at least until AI is above human-level intelligence. But competitive dynamics could prevent this from happening in certain strategically important fields.

● Might significantly speed up takeoff speed, due to sufficiently capable AI removing barriers to takeover (either takeover from misaligned AI, or aligned AI removing deployment barriers).

One important feature of this kind of deployment delay is that they increase the gap between “level of AI that has been integrated into the economy and thus empowers people, organisations, governments, etc” and “level of AI available to the much smaller group of actors who control SOTA AI”. I think it’s notable if this gap becomes very large. It massively increases the relative power of a very small group of actors.

Deployment delays that do delay future AI progress.

These deployment delays affect both capabilities takeoff speed and impact takeoff speed.

They will tend to make capabilities takeoff speed slower. For example, section 5 described the following feedback loop in software R&D:94

(A) Better software → train better AIs → [delays] → automate more tasks in software R&D → better software

If there are delays to deploying AIs in software R&D then this feedback loop will take longer to build up momentum, software progress will be slower, and we’ll take longer to cross the effective FLOP gap.

How big would these delays be? My guess is that deployment delays in hardware R&D, software R&D and chip manufacturing will be significantly smaller than in the general economy (e.g. months rather than years) because the specific candidate delays (regulations, political resistance from workers, people distrusting AI) seem less applicable in these contexts where AIs aren’t directly interacting with human customers. This is just a guess though; this is another important subquestion that I haven’t investigated.

If there are Diminishing Delays to deployment, then that could cause (both impact and capabilities) takeoff to be harder. In an extreme case, if the above feedback loop had very large deployment delays until we had AGI, but then had no further delays, then software would suddenly start increasing extremely rapidly at that time.95

94 I’ve slightly simplified this feedback loop compared to the presentation in section 5.

95 Whether this example counts as a “fast takeoff” would depend on the metric you’re using. If you’re using the serial time metric “time from AI that can perform 20% of cognitive tasks to AGI” then the delays in this example make takeoff slower. But if you’re using the metric “ratio between successive doubling times of total cognitive output” then the delays in the example might make takeoff faster: the doubling time after AGI would be very quick compared to the doubling time before AGI.
Overall, I’d guess that delays of this kind suggest takeoff will be slightly slower than the predictions of the FTM.

**Unmodelled bottlenecks to developing AI systems**

The last section considered delays that occur after “we’ve developed this AI” and before “we’ve deployed it wherever we can”. This section considers delays to developing AI systems in the first place.

The takeoff speeds framework of this report leans strongly on the view that software and compute are the key inputs to AI development. It assumes an AI is trained once enough 2020-FLOP is used to train it; and 2020-FLOP equals physical FLOP multiplied by the quality of the software.

But perhaps AI development will be bottlenecked by a lack of training data. It is often possible to use software and compute to generate data: we can apply transformations to training data to generate more data, or train in simulated environments. But, especially if human-labelled data or data about the actual physical world is required, better software and compute may be unable to compensate.

The Bio Anchors report suggests this won’t be a major issue but could cause several years of delay. If so, this bottleneck could prevent a takeoff happening in <3 years but probably wouldn’t prevent a takeoff happening in 5 - 10 years. On the other hand, if data bottlenecks are removed very suddenly, takeoff might be faster than what I’m forecasting here.

Are there crucial inputs to AI development other than software, compute and data? Bio Anchors raises the possibility that wall-clock time for training could be impractically large, but points to evidence that data parallelism allows us to scale up training runs massively, while only increasing the number of steps of gradient descent slightly or not at all. Significant engineering effort may be required hard to scale up data parallelism, but I have tried to account for this as a barrier to rapidly increasing the fraction of global FLOP used on a single training run. To my mind, this consideration speaks against takeoff happening in a few months, but not against it happening in a few years.

In addition to these comments about specific candidate bottlenecks, I have some general thoughts about how to think about inputs to AI development ignored by this framework. These thoughts apply to the above bottlenecks, and to others I haven’t discussed.

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96 See discussion from Bio Anchors.
97 Ajeya Cotra writes, “On balance, I expect that training data and environments are likely to be available roughly around the time that the requisite computation is affordable, but I do consider it plausible that data/environments will cause several years of delay, particularly if high-compute and high-data hypotheses are true.”
Most of the dynamics I’ve modelled for compute and software will apply to other inputs to AI development.

- The key dynamics I’ve modelled are i) the effect of fast rising human investment, and ii) the effect of incremental AI automation.
- These same dynamics would affect the growth of any other inputs to AI development. For example, assuming data is an important input, I expect that in the run-up to AGI there will be large human investments in gathering data and pre-AGI systems will be used where possible to gather data (e.g. AIs controlling drones that take photos).
- This makes me think that modelling these dynamics for an additional input (other than compute and software) wouldn’t change the results very significantly.

An important question here is how much AIs can help with the additional input. AIs are plausibly very well placed to perform hardware and software R&D, as they load heavily on cognitive abilities. By contrast, cognitive skills seem less important for data gathering.

- Qualifier: A key dynamic that may not apply to other areas is “these inputs are already growing very quickly”. This is true of hardware and software, but may not be true of (e.g.) data.

If you think some omitted factor will result in long AI timelines, including it would probably result in slower takeoff

- For example, suppose you think data requirements are so severe that we won’t get AGI before 2060.
- This makes it more likely that, even after we get a startpoint AI (for me this is AI that performs 20% of cognitive tasks), lots of work must still be done to get enough data to train AGI. In which case there will be more time between the startpoint AI and AGI, so slower takeoff.98

If a bottleneck could be removed quickly, it could result in a faster takeoff.

- Suppose the software and compute for AGI is in place, but another factor bottlenecks progress well below AGI. Then quickly removing that bottleneck would cause a fast increase in AI capabilities up to AGI.

Overall, my best guess is that someone who believes I’m omitting important inputs to AI development should put less probability on the fastest takeoff scenarios of the model. The fastest takeoff scenarios are the most vulnerable to being blocked by a few years of delay. But beyond this, I don’t see this as a general reason to think the framework is biased in either direction.

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98 We can think of this in terms of a generalised effective FLOP gap. We could apply the same concept to another input. E.g. we could define the “data gap” as the amount by which our data sets must improve to go from “automate 20% of cognitive tasks” to AGI. One important constraint on the effective FLOP gap is the training FLOP for AGI; it places an upper bound on the effective FLOP gap. Similarly, a constraint on the data gap would be the difficulty of getting good enough data to train AGI. If this was very difficult, the data gap could be very larger which, by analogy with the effective FLOP gap, would result in a slower takeoff.
There might be a discontinuous jump in AI capability

The majority of my probability mass is on scenarios where AI capabilities improve smoothly as we cross an effective FLOP gap whose size is 1 - 8 OOMs of 2020-FLOP. That is, I’m assuming that AI capabilities increase continuously with additional inputs of compute and software R&D.

An extremely discontinuous view of takeoff can be (hackily) represented in this framework by using a very small effective FLOP gap. With a sufficiently small effective FLOP gap, we can go overnight from “~0% of cognitive tasks automated” to “~100% of cognitive tasks automated”. With this assumption, and the assumption that you can’t trade off training compute and runtime compute, you can derive the conclusion that an intelligence explosion happens within just a few days.

This appendix discusses discontinuities in more detail, in the context of the views of Eliezer Yudkowsky and Nate Soares, and explains why I put ~6% probability on a substantial discontinuity in AI progress.

Focuses mostly on the transition to AGI rather than the aftermath

This framework is mostly geared towards forecasting the transition from today’s world to a world where AI can automate ~all cognitive tasks (AGI). It is less well placed to reason about growth in the aftermath of AGI.

Why? The task-based model represents AI progress via the ability of AIs to (collectively) perform an increasing fraction of value-weighted cognitive tasks. An improvement in AI capabilities is represented by the ability to perform additional cognitive tasks. But after we have

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99 This is encoded in the assumption that cognitive tasks are distributed fairly smoothly on a graph of log(2020-FLOP).
100 More precisely, I’m assuming “% cognitive tasks performed by AI” in software R&D, hardware R&D, GWP (tasks for directly producing goods and services) improve continuously as the 2020-FLOP used in training runs increases. And I’m also assuming that FLOP/$ and 2020-FLOP per FLOP (i.e. software) increase continuously with inputs to hardware and software R&D.
101 Recall that I’m weighting these cognitive tasks by economic value in 2020.
102 If you can trade these off then takeoff takes months even with the assumption of no effective FLOP gap. I discuss a model of this kind in an appendix.
Here’s the intuition behind the result. Suppose you can’t do the tradeoff and have loads of runtime compute lying around. Then one day you 2X your training run and go straight from “AI can perform 0% tasks” to “100% tasks” and you can run 100s millions of AGI. But if you can do the tradeoff then at some earlier time you will have been able to perform 100% of tasks by using a huge amount of runtime FLOP. Each ‘AGI equivalent’ will have taken lots of compute to run, but because you have so much compute lying around you can do it. So initially, your AGI workforce will be smaller than your human one, because it’s so compute-expensive to run. Then as your training runs get bigger, you’ll need less runtime compute and you’ll be able to run more and more AGIs. But you won’t go from nothing to millions of AGIs overnight.
automated all cognitive tasks, this way of representing AI progress does not work. There are no additional tasks for AI to learn.

I don’t think this is a significant drawback for a few reasons:

- The FTM does allow AI to perform the fixed list of tasks increasingly efficiently by doing bigger training runs after AGI, and this dynamic can help to cause explosive growth in cognitive output. Indeed, when assessing whether a software only singularity might occur, I take into account the prospect of training increasingly intelligent AGIs.103
  - Even if you think larger and smarter brains will enable AGIs to do truly amazing things, you could in principle represent this via very large efficiency benefits from doing bigger training runs after AGI.
- I think that the ultimate origin of any fast takeoff is likely to reside in the transition to AGI. This is because I think that the internal dynamics of the world after AGI alone are unlikely to lead to a fast takeoff by themselves, absent a sufficiently quick transition to that world. I explain my thinking in this appendix.
- I think things will be going pretty crazy by the time we have AGI, and this is also supported by the model outputs (e.g. >30% GWP growth, total cognitive output >100X what humans could do without AIs, extremely fast hardware and software progress). This suggests to me that the majority of the most strategically significant milestones (in terms of positively influencing the long run trajectory of AI) will be crossed before we get to AGI.

I’m not sure how modelling the aftermath of AGI in more detail would affect takeoff speeds. I can imagine some models of the benefits of “more qualitative intelligence” might have important implications that my model is missing.

Implies GWP growth is sure to accelerate eventually

The growth model I’m using here implies that developing AGI is certain to significantly accelerate GWP growth. This is the implication of nearly all growth models that represent AGI via the automation of all tasks previously performed by human labour (including R&D tasks).

However, most economists don’t believe that AGI would have this effect on growth.104 Some economists explained their reasons in reviews of my report on AI and growth. Their reasons can often be characterised as involving bottlenecks that slow down growth. For example:

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103 [Explain the FTM assumptions about training larger models after AGI in general. Before AGI, larger models can perform new cognitive tasks. After AGI, can larger models perform existing tasks more efficiently?]

104 [In the most comprehensive survey of economists to date, they assign low probabilities to significant increases in growth (more). Unpublished surveys find that, even assuming that we develop machines that can perform all economically relevant tasks more cheaply than humans, economists still assign low (<20%) probabilities to significant increases in growth (annual global growth of GDP/capita >10%).]
Historically growth has required huge physical infrastructure projects (electricity, water systems, roads). Even with billions of AGIs, there would be bottlenecks to doing these projects 10X faster.\textsuperscript{105} Even if AI increases the quality and quantity of many products, there will be some products it won’t improve quickly and these will bottleneck growth.\textsuperscript{106} One specific hypothesis here is an intrinsic preference for human-produced products that AI, by definition, can’t provide.

People don’t want new products to be introduced too quickly as it takes them a while to adjust; this limits the rate at which new products can be introduced and thus the rate of economic growth.\textsuperscript{107}

Assessing these arguments is important, but beyond the scope of this report.\textsuperscript{108} For now I will simply say that:

- To the extent you buy these objections, they are also objections to a fast takeoff. (If bottlenecks prevent growth ever accelerating, they will to an even greater degree prevent it from suddenly accelerating.)
- In general, these objections don’t seem very relevant to capabilities takeoff. They seem to target the link between high AI capabilities and impact (suggesting that bottlenecks will get in the way), without disputing the capabilities themselves. (Although impact can feedback into capabilities, as discussed above.)
- For impact takeoff, we can distinguish between fast takeoff in GWP vs other strategic areas (e.g. military power, total cognitive output, the level of SOTA technology). These objections, to me, seem much more relevant to GWP than to other strategic areas. (Justifying this claim is important, but beyond the scope of this report.)
- It’s possible the people making these objections aren’t truly imagining AI that can do all tasks as well as a person. I and others have sometimes had this impression when speaking to economists, and some economists have reported that they had previously not been conducting the thought experiment properly.

Overall, this adds some hazy uncertainty to the picture by raising the possibility that the growth model I’m using is fundamentally flawed, and this uncertainty pushes towards slower takeoff (or no takeoff at all). Personally, I’m not as moved by this kind of objection, because I don’t expect them to be correct in areas of strategic importance (military power, total cognitive output, level of SOTA technology).

I analyse why this report’s framework can predict fast takeoff in GDP, in the context of what generic growth models say about takeoff speed, in this appendix.

\textsuperscript{105} See Ben Jones’ review, starting here, for something similar to this.
\textsuperscript{106} See Dietrich Vollrath’s review, starting here, for something along these lines.
\textsuperscript{107} Phil Trammell suggests this, and the previous bullet, could bottleneck growth here.
\textsuperscript{108} My report Could Advanced AI Drive Explosive Economic Growth explains my overall take on these issues, including on various objections to explosive growth. In addition, I respond to the specific objections to explosive growth made by reviewers; I appended my responses to their reviews. I hope to do or commission more work investigating these issues.
The FTM assumes that investment in AI R&D vs other areas grows exogenously, rather than modelling actors’ incentives.

Society will allocate resources between the three buckets of hardware R&D, software R&D and buying chips for training AI. These resources will be allocated according to the incentives faced by different actors (AI labs, governments). I have not tried to model these incentives. Instead, I’ve assumed that investment rates grow at their current rate until “wake-up” and then grow more quickly. I also placed a cap on the % GDP invested in each bucket and a cap on the fraction of chips used on the largest training run before and after “wake up”.

These “exogenous” growth rates are my guess at how the incentives will fall out, informed by historical analogues. But more detailed thinking about the incentives could reveal that the investment dynamics are significantly different than in my assumptions.

This is especially likely for parameter values that are very different from my best-guess; then the investment patterns that “seemed sensible” to me in scenarios like the best-guess scenario might not hold. For example, if AGI training requirements are extremely high and there’s a large effective FLOP gap, then AI automating 6% of cognitive tasks might not cause “wake up” because it would look like a continuation of the slow pace of automation that has already been happening before AI.

- Doesn’t model actors’ incentives to invest in training runs and AI R&D in detail, but instead makes hacky assumptions about how investments change before and after the world “wakes up” to AI’s full economic and strategic potential.

Elides the distinction between capabilities takeoff speed not impact takeoff speed

By assuming immediate deployment of AI, the FTM implicitly assumes that AI’s impact moves in lock-step with its capabilities. That is capability takeoff speed is the same as impact takeoff speed. (More on the difference between these two.)

Other

There are a number of additional limitations to the FTM. Here I briefly list a few salient ones:

- Doesn’t model changes in $\rho$ (degree of substitutability between different economic tasks) over time despite the fact that short-run estimates of $\rho$ are lower (less substitutable) than long-run estimates of $\rho$ (which suggest $\rho = \sim 0$, as in Cobb Douglas).
  - I’ve erred much more towards the short-run estimates as I’m interested in the prospect for fast takeoff, but this means I may underestimate the long-run effect of AI on GWP.
• Assumes human workers are immediately reallocated to new tasks.
  □ When AI can perform a new task, FTM assumes human workers are immediately reallocated to non-automated tasks.
  □ In many contexts, this is reasonable. E.g. text completion AIs save everyone some time and allow them to spend more time on other tasks.
  □ But if AIs automate all aspects of a job, then the workers won’t have other tasks that they can spend more time on instead. In this case, there may be months or years before workers get new jobs.
  □ This limitation will lead takeoff to be slower than the predictions of FTM.
• Doesn’t model robotics. This framework focuses on the automation of cognitive tasks. But an important part of takeoff will be the way in which robotics advances alongside and complements progress in disembodied AI.
  □ If robotics keeps up, then the capital bottlenecks I’ve discussed may be less severe than the FTM is predicting. (I don’t expect robotics to keep up because the fast pace of progress in hardware and software is very unusual and so I expect the number of disembodied AGIs to double more quickly than the number of robots).
  □ Relatedly, the FTM doesn’t model physical human labour. It assumes tasks are either done by cognitive labour or physical capital. For this reason, I use an unusually high capital share (50%).
• I list some additional minor weaknesses here.

This is the end of the main body of the full report. Part 3 contains additional appendices.
One-dimensional model of takeoff speed

Summary

The one-dimensional model of takeoff measures human and AI cognitive output along just one axis. Total output = human output + AI output. This contrasts with the Full Takeoff Model, which considered AI performance and multiple different tasks.

The one-dimensional model estimates how quickly AI output will grow around the human range via there being more AIs and cleverer AIs. The estimate is based on assumptions about the rates of progress in software and hardware, and about how AI output scales with model size.

When we define takeoff as the time from “AI output is 1% of human output” to “AI output is equal to human output”, the 1-d model implies that takeoff will take ~5 months to ~1.4 years.

What is the one-dimensional model?

The full takeoff model in this report (FTM) analyses AI capabilities by asking: what fraction of value-weighted tasks can AI automate? Implicit in this is a multi-dimensional view of AI capabilities. There are many different tasks. AI is better at some tasks than others. AI can replace humans at some tasks, but not at others.

In economic models with many tasks, there are diminishing economic returns to additional output on any individual task. Total output (i.e. GDP or R&D progress) depends on i) the output on each task and ii) how different tasks combine together to produce total output.¹

[As a toy example, let’s pretend there are only two tasks that are equally important to production. The first task is performed by humans and has output H, the second task is performed by AIs and has output C. If the tasks combine together to make total output Y via a Cobb Douglas formula then:

\[ Y = H^{0.5} C^{0.5} \]

¹ Section 6 discusses Cobb Douglas and CES production functions for calculating total output from task-specific outputs. A theme is the extent to which low output on just one task can ‘bottleneck’ total output.
In the FTM there are dozens of such tasks, with the fraction being performed by AI rising over time.

I think the multi-dimensional view of AI capabilities is correct; AI is much better at translation than teaching physics.

However, you can instead model takeoff speeds using a one-dimensional view of AI capabilities. In such a model, there is only one task that creates output. As AI improves it becomes more productive (per FLOP) at this task. This model can be motivated by pointing to how training bigger foundation models improves performance across a very wide range of downstream tasks. One component, like a g-factor for AI intelligence, explains cognitive ability in diverse domains.

To calculate total output you simply sum human output at the task (H) and AI output at the task (C):

\[ Y = H + C \]

Initially, C is much smaller than H. But C grows faster than H, and by the end of takeoff it is much larger than H. C increases due to a mixture of cleverer AIs and more AIs.

When C first becomes close to H (e.g. C = 0.1H), AIs start contributing a meaningful amount to AI R&D. (This will happen earlier if the fraction of AI output directed towards AI R&D is larger.
than the fraction of human output, as I think is likely. This increases the rate of AI R&D progress and so accelerates the growth of C. Growth increases until you hit physical limits. This is shown in the dotted line above.

How does the model work mathematically?

The crucial question is: how quickly does AI output grow? In my preferred one-dimensional model, AI output depends on three things:

1. **Runtime FLOP.** The FLOP available to run AI.
   a. The simplest choice is to assume that output is proportional to runtime FLOP.
2. **Training run size.** The largest training run done to date. Bigger training runs develop more capable AIs that produce more output per FLOP at runtime.
   a. The functional form I favour is: *each doubling of training run size doubles output per FLOP N times.*
   b. This functional form has intuitive behaviour and is implied by results from ML toy experiments that independently vary runtime and training FLOP.
3. **Software.** The algorithms available for training and running AIs.
   a. As elsewhere in the report, I measure software in units whereby doubling software has the same effect as doubling the amount of hardware available for training AIs and running AIs. So doubling software is equivalent to doubling runtime FLOP and doubling output per FLOP N times.

This implies the following equation for AI output:

\[ C = (\text{software} \times \text{training FLOP})^N \text{ (software} \times \text{runtime FLOP}) \]

The first part of this expression corresponds to “how clever are AIs?”, the second part to “how many AIs can we run?”.

We can then split FLOP into $ spend and FLOP/\$: 

\[ C = (\text{software} \times \text{FLOP/}$$ \times \$$ \text{ on training FLOP})^N \text{ (software} \times \text{FLOP/}$$ \times \$$ \text{ on runtime FLOP}) \]
\[ C = \text{software} \times \text{FLOP/}$$ \times \$$ \text{ on training FLOP} \times \text{FLOP/}$$ \times \$$ \text{ on runtime FLOP} \]

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2. When we are close to AGI, I expect people to want to substantially increase investments in AI R&D compared to today. It’s hard to quickly redirect human labour to this end; it requires retraining. But AI output can be very quickly redirected towards AI R&D by simply using a greater proportion of compute to run AIs that do AI R&D.

3. Or, at least, I haven’t noticed any unintuitive consequences of the definition, unlike for obvious alternatives. If a doubling of training run size merely increased output per FLOP by a constant *absolute* amount, then there would be very sharply diminishing returns to bigger training runs; increasing training run size (and model size) would have negligible benefits past a certain point. Conversely, if each *absolute* increase in training size doubled output per FLOP, then each doubling of training run size would cause ~2X the performance improvement of the previous doubling.

4. Andy Jones (2021): “Knowing now that compute can be spent in two places, at train time and test time, the immediate question is: how do these two budgets trade off? … the trade-off is linear in log-compute”.
Then the growth rate of AI output is:

\[ g(C) = (N + 1) \cdot g(\text{software}) + (N + 1) \cdot g(\text{FLOP/\$}) + N \cdot g(\text{$ on training FLOP}) + g(\text{$ on runtime FLOP}) \]

We could additionally specify:

- How do \( g(\text{software}) \) and \( g(\text{FLOP/\$}) \) depend on the investments in software and hardware R&D?\(^5\)
- In each timestep, what fractions of human output and AI output are spent on training FLOP, runtime FLOP, software R&D, and hardware R&D?

What does this model imply about takeoff speeds?

Here I define takeoff as the time from AI output being 1% of human output to it being equal to human output.\(^6\) On my quick analysis, the 1-d model implies that takeoff will take \( \sim 5 \) months to \( \sim 1.4 \) years.

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5 As in these subsections of section 4
6 This definition is different to the main definition used elsewhere in the report, in a direction that biases the results somewhat towards slower takeoff.

The main definition used elsewhere in the report is “time from AI that can perform 10% of cognitive tasks to when we can run 10 billion AGIs”. This definition doesn’t directly translate to the 1-d model, where there’s only one task.

A rough translation to the 1-d model would be “time from AI increasing output by \( \sim 10-20\% \) to AI increasing output by \( \sim 4X \)”. (Automating 10% of cognitive tasks raises cognitive output by \( \sim 10-20\% \). If there are \( \sim 3 \) billion human workers today, having 10 billions AGIs would increase that number by \( \sim 4X \).)

So there are two definitions we could use for the 1-d model.

1. The definition I use in this appendix: \textit{time from AI increasing output by 1\% to AI increasing output by 2X}.
2. The rough translation from the main definition used elsewhere: \textit{time from AI increasing output by \( \sim 10-20\% \) to AI increasing output by \( \sim 4X \)}.

Takeoff is faster according to definition 2. Why? In the 1-d model, output doubles extremely quickly by the time AI has doubled output. (This is because a large fraction of the extra AI output can be directed to AI R&D, increasing annual R&D inputs by \( >10X \).) So the endpoints of both definitions happen at roughly the same time. But the startpoint for definition 1 is a fair bit earlier.

Why did I not just use definition 2? My reason was that, in the 1-d model, AI R&D is already accelerating by \( \sim 5X \) by the time AI has increased GDP by 10\%.\(^*\) This felt too dramatic for a startpoint.

\(^*\)This acceleration is much greater in the 1-d model than in the Full Takeoff Model. Why? In the 1-d model, AI output is a separate pot that can potentially \textbf{all} be invested in AI R&D, significantly increasing R&D inputs. Whereas in the Full Takeoff Model AI is initially complementary to humans so can only enhance the output of the small number of human AI R&D workers.)
Below I first discuss plausible values for parameters determining the growth of AI output and then apply those numbers to get conservative and aggressive estimates of takeoff speed.

Notes on possible parameter values

- **g(software)**
  - Section 4 assumed software had been doubling every 32 months (26% growth), although data from *AI and Efficiency* implies 52% growth.
  - This could be faster as we approach AGI due to people making large investments in anticipation, or slower if we cannot sustain investment growth at the current fast pace.

- **g(FLOP/$)**
  - FLOP/$ of GPUs have been doubling every ~2.5 years (28% growth).
  - I expect this to be faster as we approach AGI, as total hardware R&D investments have grown slowly at only ~4% over the last ~15 years.

- **g($ on training FLOP)**
  - $ on training FLOP = $ on FLOP globally + $ on FLOP on the largest training run.
  - Above I noted that I expect there to be ~1-3 OOMs room to scale up the fraction of FLOP on training runs by 2030, and think this fraction could potentially increase very quickly. I guessed 1 OOM every two years after “wake up”, with massive uncertainty, which implies g(fraction of FLOP on the largest training run) = 110%.
  - Above I guessed that we might increase $ on FLOP globally at ~22% (~3 year doubling time), with a range of 10% - 40%.

- **g($ on runtime FLOP)**
  - $ on runtime FLOP = $ on FLOP globally + $ on FLOP on running AIs.
  - I think the fraction of FLOP on running AIs has less room to grow than the fraction on training. Probably <1 OOM of growth by 2030, as AI chips are forecast to be $100s of billions while semiconductors total will be ~$1tr.
  - It can also grow less quickly, as the training fraction (but not the runtime fraction) can grow via using a larger fraction of AI chips for a big training run.
  - Below I assume g(fraction of FLOP on running AIs) = 0 for simplicity.

- **N**
  - Recall the meaning of this is: each doubling of training run size doubles output-per-FLOP N times.
  - With Chinchilla scaling, doubling the training run corresponds to half a doubling of model size. So we can reframe this as: each doubling of model size doubles output-per-FLOP 2N times.
  - This is a crucial input to the one-dimensional model, and the only one not discussed elsewhere in this report.
  - What does the evidence say about this?
    - In short, brain size – IQ – output correlations suggest N = ~2; toy ML experiments suggest N = ~1.
See the full BOTECs here.

Here’s a bit more explanatory detail:

- **Brain size – IQ – output correlations**
  - All kinds of things might affect the extent of variation in human performance, like amount of practice or random mutations. What we care about is: How far do you move through this range by training bigger models?
  - One way to get at this is to ask how brain size in humans correlates with IQ and how IQ correlates with productivity at downstream tasks. This should get at the “AI g-factor”, ignoring other causes of variation.
  - A bigger brain has more ‘training FLOP’, more ‘runtime FLOP’ and more ‘output’, so estimating the change in each can get you an estimate of how additional training FLOP increases output per FLOP at runtime.
  - How big is the effect?
    - I argue in a separate appendix that a 10% increase in brain volume causes a 4-5 extra IQ points. Given that brain size seems anti-correlated with neuron density, I’ll assume that 10% more brain FLOP/s increases IQ by 5.
    - A Garrett Jones article notes that an IQ point is associated with a ~6% gain in productivity across countries.\(^7\)
    - The sheet shows these assumptions imply that a doubling of human brain FLOP/s would double output ~3 times.
    - This is ~2 doublings of output per FLOP at runtime.
    - I’ll also assume doubling human brain FLOP/s doubles training FLOP. Why? When humans have bigger brains they still receive the same amount of ‘training data’ during their lifetime learning, so brain FLOP/s is proportional to training FLOP.
    - So we get ~2 doublings output per FLOP from 1 doubling of training FLOP: \(N = ~2\).

- **Toy ML experiments**
  - Andy Jones (2021) trained AlphaZero to play Hex, independently varying the training and runtime compute by changing the model size and the depth of tree search.

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\(^7\) This association is probably not all causal due to common causes of high average IQ and high national productivity. On the other hand, the 6% averages across all jobs but the causal impact of IQ on productivity is probably much stronger in the most cognitively demanding roles (e.g. research). Some of these roles (e.g. automating AI R&D) are particularly relevant for takeoff speeds.
- He found that “for each additional 10X of train-time compute, about 15X of test-time compute can be eliminated”.
- This implies that each doubling of training FLOP doubles output per FLOP 1.2 times. \( N = 1.2 \).

### Other sources of evidence?
- **ML scaling in narrow domains.** It should be possible to look at how increased training compute improves performance metrics in narrow domains like Go and Chess, map the performance metrics to standard deviations of human ability, map this to standard deviations of IQ (1 sd = 15 IQ points), and then map this to increases in output as above.
- **Observed training-runtime tradeoffs in LMs.** Language model performance can be improved significantly merely by increasing runtime compute, e.g. by chain-of-thought prompting or by generating multiple solutions and choosing the most common. You could quantify how much extra runtime compute corresponds to more training compute (via training a bigger model). This would essentially be repeating the toy ML experiments in the context of LMs.
- **Cognitively loaded domains.** If output scales very steeply with IQ in certain key domains that will be important for AI takeoff (e.g. software engineering), that gives us reason to use a larger value for \( N \).

### Calculating AI takeoff speed

The following table shows the doubling time for AI output from conservative inputs and aggressive inputs.

- The conservative inputs take recent growth rates (discussed above) and assume there’s no longer room to increase the fraction of FLOP in the largest training run.
- The aggressive inputs incorporate a speed-up due to rising investment (different measurement in the case of software). Note that the growth of the fraction of FLOP on the largest training run is still not that aggressive.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Conservative</th>
<th>Aggressive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of FLOP on largest training run</td>
<td>Not growing.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>----------------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>$ on FLOP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FLOP/$</td>
<td>2.5</td>
<td>2</td>
</tr>
<tr>
<td>Software</td>
<td>2.5</td>
<td>1</td>
</tr>
<tr>
<td>N Double training FLOP → double output per FLOP N times</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>AI output</td>
<td>5 months</td>
<td>1.5 months</td>
</tr>
</tbody>
</table>

If we define AI takeoff as the time from AI output being 1% of human output to it being ~equal to human output, that will be ~7 doublings which is ~3 years for the conservative case and ~10 months for the aggressive case (calcs).

This gives us the growth rate of AI output before accounting for AI accelerating AI R&D (i.e. hardware and software R&D). If we take this into account, then things will be quicker.

By the time AI output is 1% of human output, it will plausibly already be noticeably accelerating AI R&D. How much depends on the relative fractions of AI output and human output invested in AI R&D.\(^8\) If 1% of human output is invested but 50% of AI output, the speedup in AI R&D is ~1.5X. Then once AI output is 10% human output, the speedup in AI R&D is ~6X.

I very hackily assume you get a 1.5X speedup in AI R&D while AI output grows from 1% to 10% of human output, and a 6X speedup while it grows from 10% to 100%. The result is that the time from 1% to 100% goes to ~1.4 years in the conservative case and ~5 months in the aggressive case (calcs row 14).

This is super-hacky in a few ways: ignoring delays printing new chips, estimating the speedup at two points in the curve, ignoring AI effect on chip production, ignoring physical capital as an input to production. But it still gives a rough sense of the speed-up: it ~halves the time for AI takeoff. (Though the result would be different if you thought a larger fraction of human output, or a smaller fraction of AI output, would be invested in AI R&D.)

What are its strengths and weaknesses?
Strengths:

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\(^8\) AI software R&D is currently in $10s billions, hardware R&D is ~$100b; even significant increases would leave their total at ~$1tr, which is ~1% of global GDP.
Diverse cognitive abilities are correlated in humans, as captured by the g-factor. Perhaps AI will be the same.

Indeed, scaling up foundation models improves performance at diverse downstream tasks.

This model is well placed to interpret evidence about brain size – IQ – output correlations and evidence about trade-offs between runtime and training FLOP.

Particularly suitable if you expect AI to develop a “core of general intelligence” at some point and then for us to scale up that core by increasing training FLOP. The hypothesis of a “core” seems to imply an important one-dimensionality to AI capabilities.

Also suitable for thinking about time for AI takeoff in specific narrow domains like cybersecurity, nanotech, persuasion.

Weaknesses:

- Inconsistent with historical data on AI value-add.
  - AI value-add to the economy has not been doubling every 1 - 5 months, as implied by this model. In fact, the data I’ve seen have a doubling time of ~2 years.

- Ignores task dispersion.
  - AI gets good at some tasks long before it gets good at others.
  - This explains AI’s value-add today! It’s much better at ad recommendation, translation, ranking articles, etc. than it is at other tasks.

- AI takes decades to move through the human range on narrow-ish tasks like Chess, Go, translation, digit recognition.
  - The model implies it should take ~1-3 years to move through 4 standard deviations of human performance in a domain (calcs row 23).
  - I’m not sure how much of a tension this is:
    - I don’t know how quickly AI investments were increasing during this period. Perhaps the FLOP used to develop and run chess and Go engines was ~constant during this time. Perhaps research in these areas was ~neglected for decade-long periods.
    - I haven’t dug into the details of how long it took to move through a standard deviation of human performance, making a clear comparison hard. E.g. if it took 12 years for AI move through 8 standard deviations, that’s consistent with the upper end.
    - You could try to remove the discrepancy by arguing that the scaling of AI performance with model size will be better around human level than it is today. E.g. due to the human range unlocking new kinds of capabilities, or ‘agents’ scaling better than AI tools, or us discovering architectures that scale better with additional compute. This would reconcile the more aggressive brain size – IQ – output correlation evidence with the more conservative evidence from AI playing Chess and Go.

- Ignores many relevant factors like physical capital as an input to production (more).
Arguments for a “kink” in underlying capabilities

Background

The full takeoff model in this report assumes that AI capabilities progress is “smooth”. As training FLOP increases, AI gains the ability to perform more and more tasks and there are no significant kinks in that curve. Like this:

It is striking that you can still get pretty fast takeoff out of this framework, when parameters are such that the rate of continuous improvement is sufficiently fast.

The one-dimensional model of takeoff, discussed in an appendix, is similar in being fully continuous + smooth yet still predicting fairly fast AI takeoff.

Clearly though, another possible source of fast takeoff is that the underlying relationship between “inputs to AI” and “AI capabilities” is not smooth. Like this:

In this example the input is training FLOP. When training FLOP exceeds 1e34 there is a “kink” and AI capabilities improve significantly more rapidly than before. There need not be a discontinuous jump to a new capability level, nor need there be a discontinuous transition to the new regime. All that’s required is that fairly suddenly the rate at which AI capabilities improve with additional inputs starts to increase much more rapidly.

And, of course, the input to AI need not be “more training FLOP”. It could be that a new AI architecture is invented such that standard AI software development using this new architecture
leads to much faster progress. You could replace “log_{10}(FLOP)” with “software development
effort” on the x-axis above.

If there is a kink like this in AI progress, that would give an additional reason to the others in this
report to expect fast takeoff. It can be parsed as a reason to expect a very narrow effective
FLOP gap (or, for the one-dimensional model, as a reason to expect N to suddenly become very
large), but it doesn’t have to be.

Argument based on the chimp-human transition

In my mind, this argument is naturally framed as a reply to Paul Christiano’s rebuttal of an older
argument for fast takeoff. So I’ll first back-up and summarise my understanding of that original
argument and Paul’s reply.

Old chimp-human transition argument for fast takeoff

Humans’ cognitive abilities generalise way further than those of chimpanzees. We discovered
general relativity and flew to the moon despite our brains being ‘optimised’ to hunt in the
savanna. And humans evolved these cognitive abilities in just a few million years, which is very
quick on the timescale of evolution.

So somewhere between chimpanzees and humans there is a large kink in the rate at which
cognitive abilities accrue with additional inputs.

My understanding of Paul’s reply

Yes, humans are much better at science than chimpanzees. And yes, they gained these abilities
remarkably quickly on the timescales of evolution. But that doesn’t mean there was a kink in the
rate at which cognitive abilities accrue with additional inputs.

Another explanation is that evolution was not trying to get chimpanzees to do science.
Chimpanzees [or similarly clever apes] could have been pretty good at science, if evolution had
optimised them for that. They have much of the cognitive infrastructure necessary for doing
science, it’s just not pointed towards that goal.

Given that this cognitive infrastructure was in place already, it’s not that surprising that a few
small tweaks to humans makes them much better at science. All evolution had to do with
humans was redirect that cognitive infrastructure somewhat towards the task of science.

Indeed, the main reason humans are much better than chimpanzees at science is that we more
efficiently accumulate knowledge over generations, learning from each other. Once some useful
knowledge had accumulated, there was strong evolutionary selection pressure on proto-humans
to make them better at flexibly learning from their peers (e.g. via language). This selection
pressure redirected their existing cognitive infrastructure towards skills (e.g. flexible learning, accumulating culture) that make them better at science. In addition, there were then cultural evolutionary pressures towards cooperating and innovating. Groups with these behaviours were successful and so the behaviours became more common, either via expansion or via others copying the behaviours. So there were both evolutionary and cultural pressures redirecting humans’ cognitive abilities towards cultural accumulation and science.

So the “sudden jump” in human science ability was caused by evolution initially not optimising them to do science, but then evolution and culture later optimising strongly for science. By contrast, AI researchers will be specifically optimising AIs to do science (and other important cognitive tasks) throughout the R&D process. So there won’t be an analogous situation where AIs have lots of cognitive infrastructure that could be used for science but isn’t.

Here’s an analogy. Until 2020, the world had vaccinated very few people for corona viruses. But as of July 2022 almost 5 billion people have been vaccinated. Did we hit a “kink” in the ease of producing additional vaccines with further effort? No. We just redirected our pre-existing capacity towards the new goal of manufacturing and administering vaccines. In an analogous way, humans’ pre-existing cognitive infrastructure was redirected towards accumulating culture and doing science. But there was no “kink” in their cognitive abilities with additional inputs. But with AI, it will be as if we had always been producing as many vaccines as we could and so there won’t be the same sudden increase in vaccine production.

New chimp-human transition argument for fast takeoff

I heard this argument from Nate Soares, and he gets full credit for what is good in the ideas below. He has reviewed this summary, but I doubt that he would fully endorse it.

The above objection was that evolution wasn’t optimising chimps for science, and so it’s no surprise if humans quickly became good scientists. There was a mismatch between “evolution’s goal” and “the thing we evaluated humans on”.

Let’s remove this mismatch. Suppose that evolution had in fact been selecting animals for maximum transport speed, and then let’s evaluate humans on this same metric. (We could use something other than maximum transport speed and the argument would still go through.)

Evolution would have created cheetahs and falcons and faster animals still. There’s a good chance it would never have stumbled upon humans. But if it had explored widely enough to do so, it would have seen humans going from “way slower than the fastest animal” to “orders of magnitude faster than the fastest animal” in the blink of an evolutionary eye. In a mere few hundred years – compared to the millions needed to evolve new species – humans would develop trains, planes and rockets.

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If we drew a graph of time vs maximum observed transport speed, it would have a very significant kink in it at the point at which humans first had the fastest transport speed. Before the kink progress was driven by biological evolution changing genes to help animals fly faster etc. After the kink progress is driven by the cultural evolution of human civilisation, more specifically by R&D into planes and rockets etc. Humans overtake the other animals because they stumble upon cultural evolution, which is ultimately a much faster method for accumulating capabilities than biological evolution.

The analogy for AI development is that people will be optimising AIs for being personal assistants, making money, doing science research etc. The trends in SOTA performance at these tasks may all be smooth. But then, in the blink of an eye compared to the previous rate of improvement, a new approach to AI will blow these trends out of the water. This new approach will go from “much worse than SOTA” to “orders of magnitude better than SOTA” in a tiny fraction of the time previously needed to significantly improve SOTA.

Again, if we draw a graph of time vs SOTA performance, it would have a very significant kink at the point where the new approach to AI first becomes SOTA. Continuing the analogy, before the kink progress was driven by the standard methods of improving AI: gradient descent on bigger and bigger models, better data sets, new tricks like chain-of-thought prompting. But after the kink progress is driven by an entirely new process which increases AI capabilities much more rapidly.

Call this new process X. In the analogy, X is to the standard methods of improving AI as cultural evolution is to biological evolution. Perhaps X is a new learning rule, perhaps it is AIs actively optimising their cognitive resources, perhaps it’s enhanced coordination between different AIs, perhaps something else entirely. Importantly, X need not be AIs doing AI R&D or recursive self improvement - this isn’t what happened with humans.

<table>
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<th>Hypothetical evolution</th>
<th>AI development</th>
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<td>Transport speed</td>
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<td>Later source of improvement</td>
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<tr>
<td>Key outcome</td>
<td>Kink in transport speed when humans first travel faster than other animals.</td>
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</table>
Objections and replies

*I'm trying to steel man the objections and the replies here, but Nate Soares may not endorse what I say on behalf of his argument here.*

I'm suspicious about an argument based on a 'hypothetical' evolution

**Objection:** This argument is based on what *would* have happened *if* evolution had had the goal of maximising transport speed. You claim the result that humans would have suddenly blown other animals out of the water, but we can't be confident in this when we've never actually *seen* evolution do this. We haven't even thought about what might happen in much detail.

**Reply:** It's just very obvious that if evolution with this goal had developed humans they wouldn't have spent thousands let alone millions of years going from “as fast as the fastest non-human animal” to “orders of magnitude faster”. And this is enough to establish the key point: transport speed would have a major kink.

If we take the analogy seriously, we won’t discover X (the new process for improving AIs)

**Objection:** If evolution had been optimising for transport speed, humans probably just wouldn’t have evolved at all. They’re not fast, and the attributes they have that ultimately enable cultural evolution and technological progress aren’t very helpful for transport speed in the short term.

This analogy suggests that even if some process X exists which is much better at improving AIs, we won’t find it and so there will be no kink in practice.

**Reply:** Humans will have more foresight than evolution, so might easily find X even if evolution wouldn’t have.

Human progress is hyperbolic in your hypothetical scenario, so you’d see the kink coming

**Objection:** Your hypothetical evolution contained a kink because you were graphing the fastest animal over time. But if we graphed time vs *human* maximum transport speed, it would be smooth and hyperbolic.

As levels of human coordination and technology grew continuously, they’d find better and better ways to travel quickly. (Remember, they’re literally being optimised by evolution to travel quickly in this hypothetical.) If you were looking at that graph, you’d see the kink coming long in advance.

Indeed, the thing evolution *actually* optimised for was (something like) population size; and human population size did grow smoothly and ~hyperbolically.

So by analogy, if you graph the capabilities of the new approach to AI, they will look hyperbolic and so you’ll see your kink coming a long way off.
Reply: Firstly, smooth trends in SOTA performance in benchmarks is exactly the kind of thing slow takeoff people like to extrapolate; if they contain a massive kink that will be very significant.

Secondly, I’m not so sure you’d see a smooth trend in humans (e.g. see here). Some of the cognitive skills that later cause transport speed to grow very quickly won’t have short-term effects on maximum transport speed. E.g. language, mathematics, or scientific institutions. Sure, you might be able to look back and say “humans were continuously accruing capabilities that ultimately allowed them to build a rocket”, but you won’t know to look at that trend in advance.

Objection: To your second point, I think language, mathematics and scientific institutions would all increase maximum transport speed in the short term. Language by improving coordination, and maths and science by improving humans’ ability to design faster modes of transport. And remember, within this hypothetical we should be imagining that humans are at every stage trying very hard to use all their tech to travel as quickly as possible (which isn’t true in the actual world).

Reply: Even if you have 10,000 years of warning, that’s still nothing compared to the timescales of evolution, where significant changes often take a million years. And even if some people do see it coming because they’re looking in the right places, most people won’t.

Objection: We’d have more warning than 10,000 years. Humans took about a million years to evolve into a species that could do significant amounts of cultural evolution,¹⁰ a normal amount of time for big evolutionary changes. During that time you’d see their rate of improvement at transport speed (or whatever else they’re being optimised for) increase as they feel the initial effects of cultural evolution.

By analogy, it would take a few years to initially develop AI that can improve significantly via the new process X, as “a few years” is the normal time for significant improvements in AI. During this time you’d observe the rate of AI progress increase as X begins to take hold.

The kink in your hypothetical happens because evolution was hideously inefficient; the AI development process will be much more efficient.

Objection: What’s the core reason that humans can improve transport speed so much faster than other animals? It’s that biological evolution makes hideously inefficient use of its resources (animals and their environment). Each generation, it spends almost all these resources each generation doing an absurdly long and expensive test of how random genetic changes affect transport speed.

¹⁰ E.g. the first control of fire was ~2 million years ago, which is believed to have started a process whereby humans became increasingly dependent on culturally accumulated knowledge.
Human R&D gains orders of magnitude efficiency on this by having a person use a good fraction of their brain power (~1e15 FLOP/s) intelligently designing faster modes of transport. A lifetime of this work is many OOMs more effective at increasing transport speed than simply using that person’s life to test how one random genetic change affects transport speed.

So, the kink in transport speed in your hypothetical happens because evolution is hideously inefficient, so cultural evolution can be many OOMs better.

But the AI development process won’t be comparably inefficient. If we’re spending lots of time and money to train AIs on some benchmark, we will work hard to ensure the training resources are used efficiently to improve AI performance. For any candidates for X we can think of, we’ll already be building them into the AI development process. So there simply won’t be room for some new process to increase the rate of AI improvement by as many OOMs. In which case, there won’t be a comparable kink in AI capabilities.

To summarise, evolution was never like “how can I make this optimisation process more efficient?”. So it’s not surprising that it stumbled upon a way more efficient process: cultural evolution. But we are trying very hard to make AI development more efficient, so we’re less likely to stumble upon a new process with comparably large efficiency gains.

Reply: AI development could be more efficient than evolution and there still be efficiency gains of comparable size to the efficiency gain of human R&D over evolution. Gradient descent is not much smarter than evolution, and seems a long way off from an optimal method for increasing AI capabilities. Just as R&D made much more efficient use of human neurons, the new process X will use the AI’s cognitive resources much more efficiently to improve its capabilities.

Objection: But none of the candidates for X (better learning rules, better AI-AI coordination, managing your own cognitive resources) seem to involve similarly large efficiency gains. They don’t involve an entirely new kind of process driving improvements in the same way cultural evolution is a completely different kind of process to biological evolution.

Reply: First, all those candidates seem like they could involve big efficiency gains. Secondly, the failure to easily imagine candidates for X is weak evidence that X doesn’t exist. Aliens wouldn’t have predicted that cultural evolution would occur, had they been looking down on earth 2 millions years ago.

My overall take

This argument does update me towards C=“maybe some new AI technique will be developed over the course of a few months and cause AI capabilities to improve OOMs faster”.

But the update is relatively small (this feels like evidence i’m ~2-3X as likely to see in worlds where C is true):

• We haven’t actually observed ‘hypothetical evolution’ so don’t know what would happen.
● Even if we had, it’s just one example so provides limited evidence.
● Evolution is different from “the process of AI R&D” in some important ways. (They would be much more analogous if AI R&D simply consisted of one massive gradient descent training run. I flesh this out in the final objection, which I find pretty convincing.)
● This line of argument can be interpreted as “evidence for an extremely narrow FLOP gap”, but the evidence seems more speculative and indirect than the numerous other sources of evidence I considered. So it doesn’t seem like this should substantially shift my probability distribution over the FLOP gap.