On the Structural Consistency of Preferences

Aly El Gamal
ECE Department and Department of Mathematics
University of Illinois at Urbana-Champaign

Abstract

We review state of the art approaches in designing a model of human preferences that is faithful to the choices made by subjects in conducted experiments, and examine the properties of the models reached by each of the reviewed approaches. The first approach is a heuristic decision making model, where it is assumed that each subject has a list of prioritized reasons, and a satisfaction criterion for each item in the list. The reasons are searched sequentially in order of priority, and the search stops whenever a satisfaction criterion is met for the considered reason. This model is called the *priority heuristic* and aims at supporting the intransitivity of human preferences. The second model is a probabilistic strict weak order model, where a varying preference model is considered. This model is called the *random utility model* and aims at dissociating the variability of preferences from their structural inconsistency.

Index Terms

Priority Heuristic, Distribution-free Random Utility Model, Strict Weak Order Model, Strict Weak Order Polytope

I. Introduction

A binary relation is said to define a strict weak order if it satisfies two properties, asymmetry and negative transitivity. A relation $\succ$ is asymmetric over elements of a set $\mathcal{S}$ if and only if the following holds,

$$\forall a, b \in \mathcal{S}, a \succ b \Rightarrow b \nsucc a, \tag{1}$$

and is negatively transitive over elements of $\mathcal{S}$ if and only if the following holds,

$$\forall a, b, c \in \mathcal{S}, (a \nsucc b) \wedge (b \nsucc c) \Rightarrow a \nsucc c. \tag{2}$$

We now note that (1) and (2) imply the following property of transitivity,

$$\forall a, b, c \in \mathcal{S}, (a \succ b) \wedge (b \succ c) \Rightarrow a \succ c \tag{3}$$

The question of whether human choices follow a transitive preference model, has been discussed in the literature for over three decades. In [1], an argument, supported by experimental results, was introduced to suggest the intransitivity of our preferences. Among other attempts, the *priority heuristic* of [2] and the *regret theory* of [3] aim at providing explanations for this violation of transitivity. However, it is not clear whether the models behind all these theories are true. The same author of [1] also contributed to the development of the *cumulative prospect theory* in [4] and [5], that suggests the modeling of our preferences through a numerical utility function, and hence, suggesting the satisfaction of the transitivity property of (3).

In [6] and [7], the authors introduce a more relaxed probabilistic model based on numerical utilities. The key idea is that the variability of choices can dissociated from the inconsistency of preferences through a model that allows these preferences to change over time. These changes can be due to either uncertainty or hidden factors. Empirical probabilities induced from sample data are said to perfectly fit the model if a corresponding vector of probabilities lies inside a convex polytope that is constructed from the consistency constraints. The falsifiability of the introduced model is demonstrated through the tiny size of the considered polytope, and hence, the fitting results in [6] and [7] are found to be surprising, and suggesting the usefulness of the considered probabilistic approach.

A. Document Organization

In Section II, we review the priority heuristic model, through which the authors attempts to explain the intransitivity of preferences. We then introduce the random utility model in Section III, that studies the variability of choices through a probabilistic strict weak order model. We finally provide concluding remarks in Section IV.

II. The Priority Heuristic

In this section, we review the heuristic rule introduced in [2] as an attempt to find an explanation for observations and experimental results that may suggest the violation of structural consistency in human decision making. The rule introduced in [2] is called the *Priority Heuristic*, and is based on the assumption that we make our decisions based on a list of reasons with preferential ordering. There is a satisfaction criterion attached with each of the reasons. The suggested hypothesis is that whenever a person is asked to choose between two options, he or she searches the list of reasons from highest to lowest priority, and the search stops whenever the satisfaction criterion is met for the considered reason. Without loss of generality, we restrict the discussion to choices between two monetary gamble options with no losses. There are four considered reasons: the minimum gain, the maximum gain, and their corresponding probabilities. We then have 24 possible options for the preferential ordering of the priority list. Since the two probabilities are complimnetary, we only have three reasons, and the number of possible orderings is six. We now discuss the chosen ordering and support the choice by references to previous work.

It is suggested in [8] and [9] that the impact of outcome values overrides that of probabilities in our decision making process. The authors in [8] even suggest that people may completely neglect probabilities in some cases. A typical example that is used as an evidence for the primacy of outcome values over probabilities, is the lottery. When buying a lottery ticket, people tend to be driven by the amount of possible gain and ticket price, rather than the tiny probability of winning. It is for these reasons that in [2], the authors assume that the first priority rule has to be an outcome value. The six options for the ordering of the priority list now reduce to four, two options for choosing the first rule, and two options for ordering the remaining rules.

In order to reduce the number of options for possible orderings of the priority list from four to two, we need to decide whether the first considered reason is the value of the minimum gain or that of the maximum gain. It is suggested in [10] that people are risk averse in the gain domain, as they tend to prefer the lesser but more probable outcome. For example, it is intuitive to think that many people in today's american middle class would prefer a sure win of thousand dollars than a win of ten thousands that takes place with probability 0.1, even though both gambles have the same expected value. It is worth noting that this same idea of failure avoidance was incorporated in Regret Theory [3], and Disappointment Theory [11].

Now, that we have fixed the minimum outcome at the top of the priority list, we still have two possible orderings, based on the ordering of the maximum gain value and the probability of the minimum gain. The authors in [2] conducted experiments where the value of the minimum gain is fixed across the alternatives, and the probability of the minimum gain always suggests a different choice than the value of the maximum gain. One example experiment asks people to choose between a gamble, in which one is expected to win $(\text{US}\$600)$ with probability 0.5, and otherwise win nothing, and another gamble, in which one is expected to win $(\text{US}\$3000)$ with probability 0.1, and otherwise win nothing. The majority of people prefer the first gamble because of the higher probability of winning, i.e., lower probability of minimum gain. The tendency to prefer the probability of the minimum gain to the value of the maximum gain holds in other experiments conducted by the authors of [2], as well as in [12, Study 5].

Based on the above argument, the priority list suggested in [2] has the value of the minimum gain on its top, followed by the probability of the minimum gain, and finally the value of the maximum gain. We now discuss the satisfaction criterion for each of these reasons. The hypothesis that people stop the pursuit of reasons when an alternative surpasses an aspiration level was discussed in the theory of *satisficing* [13]. Our question now is about this aspiration level. In [14], it is suggested by introduced empirical evidence that this aspiration is not fixed, but is rather dependant on the value of the maximum gain. An example to this in [2] is the following: Consider two gamble options, the first gamble offers a choice between a sure win of (US\$100) and winning (US\$200) with probability 0.5. The second gamble is similar except that the value of the maximum gain in the second choice increases to (US\$2000). It is argued that people who prefer the first choice in the first gamble, may not prefer it in the second because of the higher value of the maximum gain. This shows that while the search for reasons in the decision process stopped at the value of the minimum gain for the first gamble, it proceeded to the value of the maximum gain in the second gamble. It is intuitive to think that this dependency on the maximum gain is measure based on the cultural number system, i.e., the base-10 system [14].

Following the above argument, the authors in [2] suggest that the search is solely based on the value of the minimum gain if the difference between the minimum gains in the two offered choices is greater than $1/10$ of the maximum possible gain across the two chocies. Also, a crude estimate of 0.1 difference in probability values is set as the threshold for satisfaction when considering the probability of the minimum gain.

III. The Random Utility Model

While the priority heuristic [2] and regret theory [3] are attempts to justify the structural inconsistency of preferences, the work in [7] and [6] provides a possible justification for the seemingly inconsistent collected data and observations about our decision making process; a justification that supports the hypothesis of a variable but consistent preference model. In [7], the authors test the hypothesis of a variable linear order preference model for binary choice data. We review in this section a similar approach followed in [6] for testing a variable strict weak order preference model for ternary paired comparisons.

For a collection of choices $\mathcal{C}$ and any pair of choices $i, j \in \mathcal{C}$, the empirical probabilities P_{ij} and P_{ji} are induced from sample data taken for a single subject in the experiments conducted in [6]. P_{ij} reflects the frequency of the subject preferring choice i to chocie j. We call the collection $(P_{ij})_{i,j\in\mathcal{C}}$ a system of ternary paired comparison probabilities if and only if $\forall i, j \in \mathcal{C}, P_{ij}, P_{ji} \geq 0, P_{ij} + P_{ji} \leq 1$. The reason for the naming is that these probabilities reflect the frequencies of paired comparisons, in which the subject can prefer one choice to another or be indifferent. Let $\mathcal{WO}_{\mathcal{C}}$ be the set of all strict weak orders on the choices of $\mathcal{C}$, then a system of ternary paired comparisons is called to satisfy the *strict weak order model* if and only if there exists a probability distribution on the set $\mathcal{WO}_{\mathcal{C}}$, that assigns a probability $P_{\succ}$ to each strict weak order $\succ$, and the following holds,

$$\forall i, j \in \mathcal{C} : P_{ij} = \sum_{\succ \in \mathcal{WO}_{\mathcal{C}} : i \succ j} P_{\succ} \tag{4}$$

It is worth noting that the strict weak order model is equivalent to a *random utility model* where there exists a probability measure P, defined on a set of utility functions $(\mathbf{U}_c)_{c\in\mathcal{C}}$, such that the following holds,

$$\forall i, j \in \mathcal{C} : P_{ij} = P\left(\mathbf{U}_i > \mathbf{U}_j\right) \tag{5}$$

For a set $\mathcal{C}$ with five choices, the authors in [6] used the *Porta* software of [15] to determine the minimum set of non-redundant inequalities that defines the strict weak order simplex. In this setting, the considered vector space has 20 dimensions, corresponding to the 20 possible paired comparisons, and the simplex has 541 vertices. The obtained set of facet-defining inequalities has $75,000+$ constraints. In order to appreciate the information gain from the experimental results obtained in [6], we highlight the fact that the weak order polytope for the case where $|\mathcal{C}| = 5$ occupies 0.0005 of the space, and hence, sample data that lie inside the polytope suggest non-trivial structural consistency with the hypothesized model.

Three Gamble sets were considered in [6], each gamble set had five choices, and 30 subjects were asked to choose between pairs of choices from the gamble sets, with the option of being indifferent between any pair of choices. The data belonging to 22 subjects out of 30 for the first gamble set generated empirical probabilities that satisfied all facet-defining inequalities of the strict weak order simplex. The number of subjects whose data *perfectly fitted* the model were 20 and 18 for the second and third gamble sets, respectively. This is quite surprising as it recommends the considered model as a good candidate for explaining the variability of choices without concluding a property of inconsistency like intransitivity. Furthermore, for the sample data points that do not lie inside the polytope, a statistical goodness-of-fit function is used, with the aid of the methods from [16]. The statistical likelihood test shows only one siginificant violation of the strict weak order model for two of the gamble sets, and two siginificant violations for the remaining gamble set.

IV. Concluding Remarks

In this survey, we reviewed two opposing models of decision making. The first is the priority heuristic of [2], and supports the hypothesis that our choices violate the principle of transitivity of preferences. The second is the random utility model of [6], that introduces a probabilistic strict weak order model, based on the assumption of a preference model that varies among a set of strict weak orders. The results consistent with the model in [6] are surprising, as it is shown through a simple geometric argument that the probability of a randomly generated sample point to satisfy the model is very low.

We find the approach of [6] interesting, as it suggests the consideration of a property other than inconsistency to explain the seemingly paradoxical choices that humans make. It is intuitive to think that our preferences do change over time, either because of uncertainty or other factors that are hidden from the investigator. If the model is indeed true, it would also be interesting to understand the triggers of these changes in preferences. We also believe that the shortest distance of a sample point to a simplex vertex may be an indicator to the degree of variability of the decision process, and we suggest the study of this parameter for future work.

Finally, it is worth noting that each sample point in the results of [6] corresponds to the chocies of an individual subject. Unlike the data aggregation approach used in [2] where the preferences of different subjects are summed, this approach avoids the occurence of an aggregation paradox akin to the Condorcet paradox in voting systems.

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